MORTALITY MODELLING – LOGISTIC FUNCTION

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Abstract

Recently, aging of population has been an increasingly discussed topic. Improve in medical care means increase in the age until which people live on average. Therefore, it is increasingly interesting to monitor the mortality of people in higher ages. This mortality is gradually decreasing and it causes the population aging. The aging of the population will have an impact on the health and social system. Gradually, the system will need to be reformed to prepare for the increase in the number of elderly people and also for the decline of young people. This is also the reason why the mortality in these ages should be described as best as possible. One of the approaches used is to model mortality using analytical functions.

The aim of this article is to model the mortality of people at the age 60+. Mortality will be analyzed for the population of the Czech Republic. The latest available data about mortality will be used and for mortality modelling will be used Kannisto model. Attention will be also focused on the effects of the COVID-19 pandemic on mortality. The obtained results will be subsequently evaluated by proposed criterion – sum of weighted squared deviations.

Key words: mortality, models of mortality, logistic function, Kannisto model

JEL Code: J10, J11, J19

Introduction

Increasing levels of medical care and greater interest of people in a healthy lifestyle are just some of the reasons that lead to a reduction in mortality during a human life. As a result, people live to higher age on average. If this phenomenon is not sufficiently compensated by the birth rate, the population ages.

Population aging is one of the most debated topics in recent years. There is a discussion about how to prepare for population aging. Reforms of the social system are related to this.

In this paper, mortality will be analyzed for the Czech population. Attention will be focused on the mortality of people in ages 60+. The last available year will be 2020. That is, the year already affected by pandemic Covid-19 in individual countries.

1 Methodology

As was already mentioned, in this paper, attention will be focused on mortality of people in ages 60+. Several approaches can be used to analyze the mortality of these individuals. One of them is the use of analytical functions.

Kannisto model

Kannisto model is one of the most widely used analytical functions used for modeling mortality at ages 60+. It is a type of logistic function. This feature is expected to have a slower increase in mortality.

In this article, Kannisto model was used in shape (Kannisto et al, 1994 or Thatcher et al. 1998):

$$\mu_x = \frac{ae^{b.x}}{1+ae^{b.x}}, \tag{1}$$

where a and b are unknown parameters of the model, x is the age.

Another point of this paper is the application of the test criterion which could be used for the evaluation of obtained results. This might give us an information about suitability of concrete model (in this paper will be only about evaluation). After that might it give us an information which from used models could be better for modeling of mortality curve.

Evaluation of expected results

As the evaluation criterion will be used weighted squares of deviations (WSD) – minimization criterion. As weight will be used exposure to risk ($m_{t,x}^{(modelled)}$):

$$WSD = \frac{S_{t,x} + S_{t+1,x}}{2} \cdot (m_{t,x} - m_{t,x}^{(\mathrm{mod}elled)})^2, \tag{2}$$

where $m_{t,x}^{(modelled)}$ is modelled mortality curve according to Kannisto (K), $S_{t,x}$ is number of living at the beginning of year t and $S_{t+1,x}$ is number of living at the beginning of year $t + 1$ (or number of living at the end of year t).

Finally, the sum of WSD has to be calculated $\sum_{60}^{90} WSD$.

Sum of WSD is calculated in age interval <60; 90>. The same age interval was used for the estimation of unknown parameters for Kannisto. This criterion could be used for the evaluation of analytical function suitability.

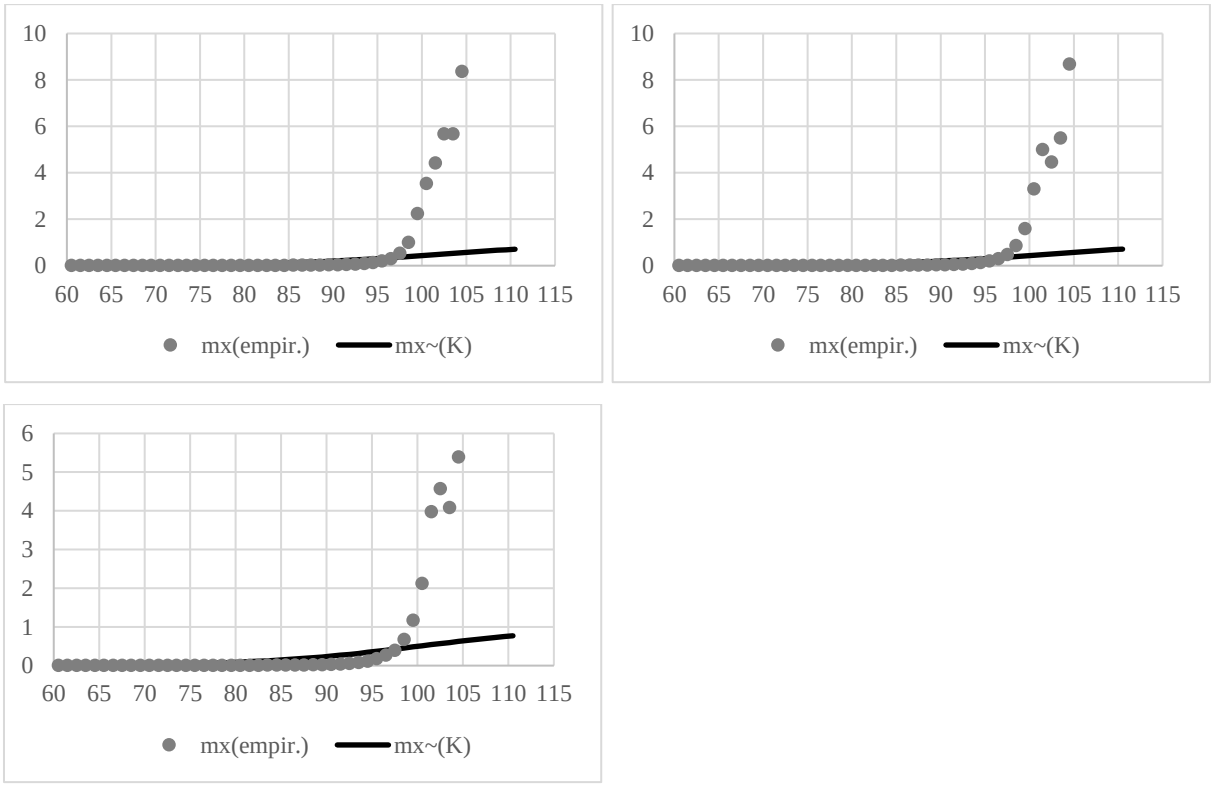
2 Results

The analysis of mortality of men and women in the Czech Republic is focused on the years 2018 - 2020. The Kannisto model was used to model mortality.

These results are also supplemented by the result of the sum of the weighted deviation counts as an evaluation criterion.

The first group of figures shows mortality of males in the Czech Republic who are in aged 60+.

Fig. 1: Modelled mortality – Kannisto - males



Source: data CZSO, author's calculation

Upon closer examination of the obtained results, we find that the Kannisto model describes well the mortality between the ages 60 and 95 years (applies to the first two graphs). However, it is important to note that the age range of 60 and 90 years was used to estimate the unknown parameters of the model. For the last graph, however, the situation is different (this is mortality in 2020). Here it is clear that problems in balancing the mortality curve occur from the age of 85. This is also confirmed by the WSD evaluation criterion used. While in 2018 and 2019 its value is in the tens, in 2020 its value is significantly higher. In 2020, data are affected

by the impact of the Covid 19 pandemic. This could have had a consequent impact on mortality modeling.

The values of the parameters of the Kannisto model and the results of the evaluation criterion are contained in Tab. 1.

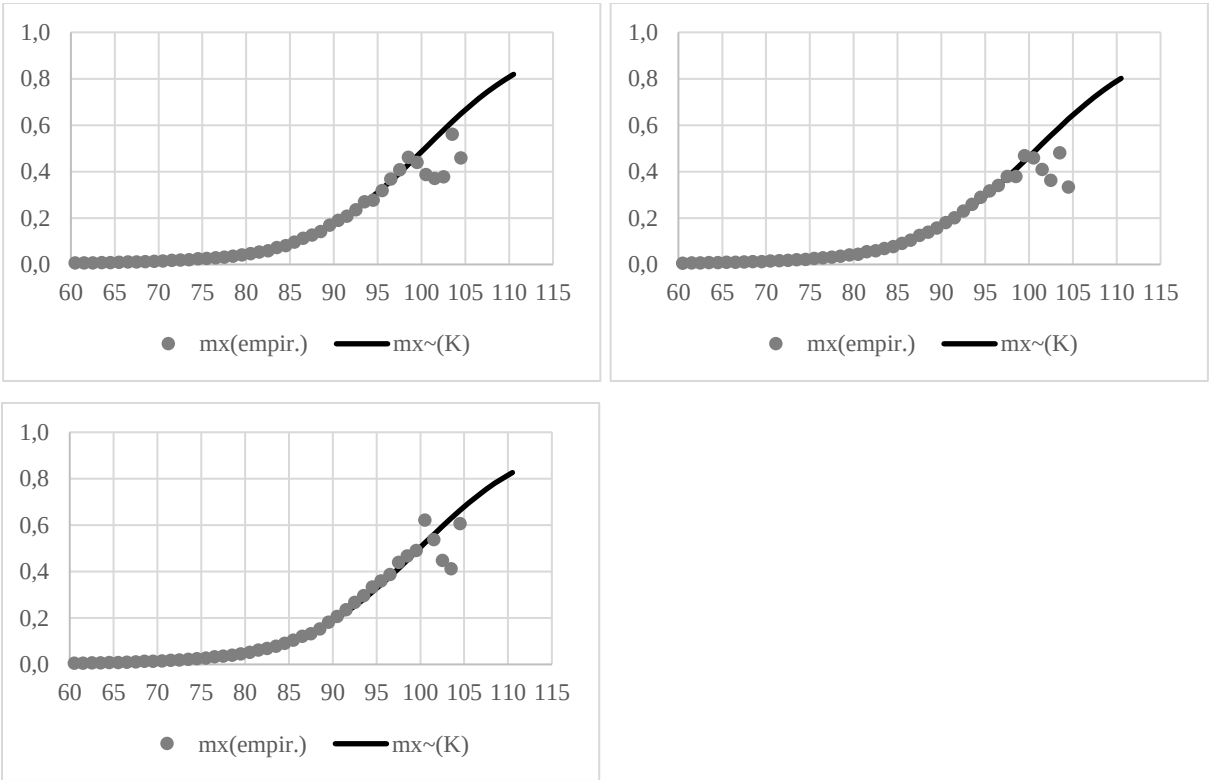
Tab. 1: Values of estimated parameters and test criterion - males

	2018	p-value	2019	p-value	2020	p-value
beta	0,000011	0	0,000010	0	0,000008	0
gamma	0,111469	0	0,112708	0	0,116627	0
R^2		0,9978		0,9979		0,9987
WSD		15,34		14,32		3616,06

Source: data CZSO, author's calculation

The second group of figures shows mortality of males in the Czech Republic in the age of 60+. Here, too, the Kannisto model was used to model mortality.

Fig. 2: Modelled mortality – Kannisto - females



Source: data CZSO, author's calculation

From the obtained results it is evident that Kannisto model describes relatively well mortality of females in the Czech Republic in the last three years. In comparison with the males' population, we find that the model works better in higher ages too than only in the selected age range (used for estimating unknown parameters). This can also be confirmed based on the WSD sum values. The results of the evaluation criterion reach (in comparison with the population of men) relatively low values in all monitored years.

The results of parameters' estimates and WSD sum values are shown in Tab. 2.

Tab. 2: Values of estimated parameters and test criterion - males

	2018	p-value	2019	p-value	2020	p-value
beta	0,000000	0	0,000000	0	0,000000	0
gamma	0,149120	0	0,146468	0	0,147219	0
R^2		0,9991		0,9991		0,9995
WSD		16,47		12,95		10,88

Source: data CZSO, author's calculation

As was already mentioned in the introduction that mortality analysis will also focus on the effects of the Covid 19 pandemic. From the modeled mortality, the effects of the pandemic are more evident in the males' population. Therefore, further attention was focused on changes in the number of deaths broken down by age units and by sex. More attention is again paid to the ages 60+.

When comparing the years 2018 and 2019, there was an increase in the number of deaths in only some cases (in addition, it is important to point out that in the highest ages the numbers of deaths are orders of magnitude lower, so information about the high increase may not be so telling).

When comparing 2019 and 2020, the situation is different. Between these years, there was an increase in the number of deaths in virtually all ages monitored.

There was also an increase in mortality among women in the "pandemic" year. In comparison 2019 with 2020, there is a noticeable increase in the number of deaths in virtually all monitored ages.

Conclusion

The aim of this paper was to model the mortality of males and females in the Czech Republic in recent years. An important part was also the study of the effects of the COVID-19 pandemic

on mortality. The Kannisto model was used to model mortality, which is also used by the Czech Statistical Office for its calculations.

In the first two years for males' population, Kannisto model describes mortality well between the ages of 60 and 95 (with an age range of 60-90 being used to estimate unknown parameters). In the last year examined - 2020, which was already more affected by the Covid 19 pandemic. This year, alignment has been problematic for about 85 years. This conclusion is also confirmed by the sum of the weighted squared deviations, which in the last year differ significantly from the result of the previous two years.

The situation is different in the females' population. In all three years examined, the alignment by Kannisto model gives relatively good results.

For completeness, changes in the number of deaths by age and sex (in persons over 60 years of age) were also monitored. Here it is evident that the increase in the number of deaths occurred in practically all ages (it was monitored only for persons 60+).

Discussion

Recently, logistic functions have been increasingly used to model mortality, which are better suited for populations with lower mortality. The Czech Statistical Office also uses logistic functions for its calculations.

Since the end of 2019 (resp. the beginning of 2020) the world has been affected by the Covid 19 pandemic. For this reason too, changes in mortality can be expected. However, the question is whether a pandemic will have a short-term or long-term impact on mortality. But mortality is likely to increase. Therefore, it will also be interesting to see if this will have any effect on the suitability of the analytical function used.

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