THE IMPACT OF SMOOTHING MORTALITY RATES ON LIFE INSURANCE

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Abstract

The contribution points to the impact of new smoothing mortality rates on the amount of premium for Endowment Insurance. It offers a comparison of the premium calculation using mortality rates published on the web site of the Statistical Office of the Slovak Republic and a smoothing mortality rates method based on so-called mixture functions. Mixture functions represent a special class of weighted average functions, where weights are determined by continuous weighting functions, which are dependent on a quotient of the original data of a number of deaths and a number of living. The advantages of this method are that the weights of the input values depend on ourselves and hence coefficients of weighted functions can be changed each year for minimization of mean square error. The effect of our method is particularly clear for single premiums in both cases. This method gives a lower probability of death, hence it can be used as a risk loading of mortality.

Key words: death probability, aggregation, moving mixture function, premium

JEL Code: G22, G28, C6, K20

Introduction

Life insurance is a contract between an insurer and a policyholder in which the insurer guarantees payment of a death benefit to named beneficiaries upon the death of the insured. The insurance company promises a death benefit in consideration of the payment of premium by the insured.

Many of clients are not aware of the presence and purpose of loading in insurance and its affect on policy premiums. According to insurers, loading is an additional cost built into the insurance policy to cover losses which are higher than anticipated for the company arising from insuring a person who is prone to a form of risk. Loading as a concept, thus, comes into play when an insurance company is dealing with a high-risk candidate, and is resorted to by insurance companies in cases where the risk to the individual is higher than in ordinary circumstances. Mortality rate is a measure of the number of deaths in a particular population, scaled to the size of that population, per unit of time. There are several different mortality rates used to monitor the level of mortality in populations. So-called crude mortality rates are most commonly used. It counts all deaths, all causes, all ages and both sexes. These data are usually available on the websites of the statistical offices of individual countries, [11], [13]. More complete image of mortality is given by life tables, which show the mortality rates separately for each age, and therefore smoothing of discrete time series and subsequent predictions of future mortality rates is a problem of fundamental importance in a demography, but also in an insurance and pension schemes, [3], [10], [15].

Our modeling is also based on relevant acts. In the Slovak Republic is valid the Act 39/2015 Coll. on insurance and amending certain laws. On the level of the European Union are in force the Council Directive 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services, the Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) and the Directive 2014/51/EU of the European Parliament and of the Council of 16 April 2014 amending Directives 2003/71/EC and 2009/138/EC and Regulations (EC) No 1060/2009, (EU) No 1094/2010 and (EU) No 1095/2010 in respect of the powers of the European Supervisory Authority (European Insurance and Occupational Pensions Authority) and the European Supervisory Authority (European Securities and Markets Authority).

This paper is organized as follows. In Section 1 we give basic notations, concepts of mixture functions and their properties. In Section 2 we introduce complete modeling of the so-called moving mixture function on smoothing of mortality rates. In Section 3 we give our results and short discussion. At the end, we give conclusions.

1 Preliminaries

In this section, we offer basic knowledge about aggregation functions and mixture functions as a subset of aggregation functions. Mixture functions represent a special class of weighted averaging functions where weights are determined by continuous, input values dependent, weighting functions. For more information see, for example [1], [2] and [6]. If they are increasing, they form an important class of aggregation functions. Because, mixture functions have a lot of very good properties, and they are also weighted averages with special weights, we want to apply a smoothing method of mortality rates just using mixture aggregation, which we investigated in our previous work, for more information see [7] and [8]. Therefore smoothing of mortality rate curves is one of the very important tools for risk management of insurance companies and represents the basic building block in all actuarial calculations [4] and [10].

Also, we recall mortality rates smoothing as a basic building block of demographic and insurance calculations. We would like to emphasize that due to standard notations, we use in this section the same notation for input values x_i, $i = 1,2,...,n$, or independent variable x, respectively (Subsection 1.1) and x as an age of an individual (Subsection 1.2).

1.1 Mixture function as an aggregation function

Throughout the paper, we give standard monotonicity of mixture functions on the interval $[0,1]$.

Definition 1. [12] (Mixture Function)

A function $M_g : [0,1]^n \to [0,1]$ given by

$$M_g(x_1, x_2, ..., x_n) = \frac{\sum_{i=1}^{n} g(x_i) \cdot x_i}{\sum_{i=1}^{n} g(x_i)} \qquad (1)$$

where $g : [0,1] \to]0, \infty[$ is a continuous weighting function, is called a mixture function.

In our approach, we consider mixture functions with three weighting functions – $g_c(x) = cx + 1 - c$, $c \in [0,1]$; $g_\gamma(x) = 1 + \gamma \cdot x^2, \gamma > 0$ and $g_a(x) = a \cdot \left(\frac{1}{a}\right)^x$, $0 < a < 1$. Because mixture function (1) need not be monotone, in general, we would like to remind you propositions about monotonicity of mixture function with the mentioned weighting functions, [8]. Mixture function (1) with the weighting function:

- $g_c(x) = cx + 1 - c$, $c \in [0,1]$, is monotone increasing for $c \in [0, 0.5]$;
- $g_\gamma(x) = 1 + \gamma \cdot x^2$, $\gamma > 0$, is monotone increasing for $\gamma \in [0,1]$,
- $g_a(x) = a \cdot \left(\frac{1}{a}\right)^x$, $0 < a < 1$, is monotone increasing for $a \in \left[\frac{1}{e}, 1\right[$.

On the basis of these mixture functions we introduced so-called *moving mixture function* and applied it in our smoothing process.

1.2 Methodology using mixture functions

The Statistical office of the Slovak Republic [11] publishes standard methodology using moving averages and we substitute just these moving averages by so-called *moving mixture function*, [8]. We give our approach as follows:

Calculation of basic probabilities. Based on available data, for entry ages x from the interval [0,105]. There we have real numbers L_x of the living at age x and D_x of deaths at age x, middle condition. Point estimate of the force of mortality at each age x is given by

$$\mu_x = \frac{D_x}{L_x} \tag{2}$$

and corresponding probability of death of an individual at age x is as follows [4], [11],

$$q_x = 1 - \exp\{-\mu_x\}. \tag{3}$$

Calculation of moving mixture function. We smooth the values q_x for the closest ages using a *moving mixture function* (4) consequently with the mentioned three weighting functions for ages $3< x< 102$. A moving mixture function is given by

$$\hat{q}_x = \frac{\sum_{j=0}^{3} g\left(q_{x\pm j}\right) \cdot q_{x\pm j}}{\sum_{j=0}^{3} g\left(q_{x\pm j}\right)} . \tag{4}$$

For probabilities obtained using moving mixture functions, we extend since age 80 the right tail by Gompertz-Makeham formula in the form $\hat{q}_x = A + B \times c^x$, [9], [14]. We look for the best intersection with moving mixture functions.

Model selection. For all coefficients of linear weighting function of moving mixture functions, we calculate Mean Square Error MSE

$$MSE = \frac{\sum_{x=0}^{105} \left(q_x - \frac{\sum_{j=0}^{3} \left(c q_{x\pm j} + 1 - c\right) q_{x\pm j}}{\sum_{j=0}^{3} \left(c q_{x\pm j} + 1 - c\right)} \right)^2 \times \left(L_x + D_x\right)}{\sum_{x=0}^{105} \left(L_x + D_x\right)} . \tag{5}$$

By the similar way, we calculate MSE using quadratic and exponential weighting functions.

1.3 Basic probabilities

The basic building blocks in modelling of all life insurance products are the relevant survival and mortality probabilities which are given as follows:

${}_t p_x$ - the probability that individual at age x survives at least to age $x+t$,

${}_t q_x$ - the probability that individual at age x dies before age $x+t$,

${}_{r|t} q_x$ - the probability that individual at age x survives r years, and then dies in the subsequent t years, that is, between ages $x+r$ and $x+r+t$.

Because our model is based on monthly benefits and we have the annual probabilities of death, we also give a fractional age assumption [4] as follows. For integer x provided the uniform distribution of deaths in every age interval $[x, x+1[$, and for $0 \le s \le 1$, we assume that

$$ {}_s q_x = s \times q_x . \tag{6} $$

On the basis of Technical notes of the European Central Bank we applied formula of the Svensson yield curve and yields of AAA rated bonds on 4 May 2018, [5].

2 Modeling of the selected product – Endowment insurance

We apply our approach to smoothing of mortality rates on basic insurance product – Endowment insurance.

Firstly, we give basic notations:

- IS - sum insured as an absolute amount in monetary units (hereafter euros),
- $R(z)$ - yield from a risk-free bond investment with continuous compounding (% p.a.),
- $P(z)$ - discounting factor, where

$$ P(z) = \exp\left\{ - \frac{R(z)}{100\%} \times z \right\}, \tag{7} $$

- x - age at entry,
- ω - maximum age to which a person can live to see (regarding used life tables is here $\omega = 105$, [11],
- α - initial costs as a % from a sum insured,

- β - administrative expenditures as a % from a sum insured payable monthly,
- γ - administrative expenditures as a % from the yearly premium,
- IC - initial costs as an absolute amount in monetary units.

2.1 Monthly premium

Under a term life insurance policy, the death benefit is payable only if the insured dies within a fixed term of, say, n years. We consider the situation when a death benefit of 1 monetary unit is payable at the end of the month of death of the insured.

We recall basic formulas on calculation of Monthly paid Premium for *Term Life Insurance (TLI)* for x aged individual who will be insured on n years. Monthly paid premium in this case is given by

$$(TLI)MP_{xn} = \frac{IS \times \left(A_{xn}^{1(12)} + \frac{\alpha}{100\%} + \frac{\beta}{100\%} \times \ddot{a}_{xn}^{(12)} \right) + IC}{12 \times \ddot{a}_{xn}^{(12)} \times \left(1 - \frac{\gamma}{100\%} \right)}, \tag{8}$$

where

$$A_{xn}^{1(12)} = \sum_{t=0}^{12\times n-1} {}_{\frac{t}{12}\left|\frac{1}{12}\right.} q_x \times P\left(\frac{t+1}{12} \right) \tag{9}$$

is the expected present value of the benefit and

$$\ddot{a}_{xn}^{(12)} = \frac{1}{12} \times \sum_{t=0}^{12\times n-1} {}_{\frac{t}{12}} p_x \times P\left(\frac{t}{12} \right) \tag{10}$$

is the expected present value of the basic premium in the amount of 1/12 of monetary unit.

Moreover, we give formula on calculation of Monthly paid Premium for *Pure Endowment insurance (PE).* Pure endowment benefit is conditional on the survival of the insured. In this case, a survivor benefit of 1 monetary unit will be paid out if the insured will be alive at age $x+n$. The formula is as follows.

$$(PE)MP_{xn} = \frac{IS \times \left(A_{x\overset{1}{n}} + \frac{\alpha}{100\%} + \frac{\beta}{100\%} \times \ddot{a}_{xn}^{(12)} \right) + IC}{12 \times \ddot{a}_{xn}^{(12)} \times \left(1 - \frac{\gamma}{100\%} \right)}, \tag{11}$$

where $$A_{xn}^{\;1} = {}_{n}p_{x} \times P(n) \qquad (12)$$

is the expected present value of the mentioned benefit.

2.2 Analysis of smoothing curves

In our investigation, we use sum insured in the amount of 10,000 €, age at entry x=47 years, n=20 years, initial costs α in the amount of 3 % from a sum insured, *IC* in the amount of 300 €, administrative expenditures β 0.1 % from a sum insured and gamma 2.4 % from a sum insured. Particularly in this setting costs, we received almost identical value of premiums, as we have gained from real life insurance company. Moreover, on the basis of formula (5) for MSE we found out that the best a moving mixture function will be function with the exponential weighting function and parameter $a = 0.683959$.

In Table 1 are written individual probabilities of death with respect to an entry age which are published by the Statistical office of the Slovak Republic and which are set by our approach.

Tab. 1: Probabilities of death q_x w.r.t. the Statistical Office of the Slovak Republic and set by moving mixture function with weighting functions

x	Statistical Office of The SR	$g_c(x) = cx + 1 - c$	$g_\gamma(x) = 1 + \gamma \cdot x^2$	$g_a(x) = a \cdot \left(\frac{1}{a}\right)^x$
47	0.003173867	0.003123030	0.003122897	0.003123052
48	0.003425510	0.003489645	0.003489531	0.003489663
49	0.003758658	0.003738489	0.003738403	0.003738503
50	0.004155526	0.004133731	0.004133501	0.004133769
51	0.004529290	0.004600240	0.004599832	0.004600308
52	0.005183851	0.005185272	0.005184728	0.005185363
53	0.005842840	0.005721828	0.005721280	0.005721920
54	0.006432884	0.006431818	0.006431061	0.006431946
.	.	.	.	.
.	.	.	.	.
100	0.489566716	0.399121405	0.393994886	0.392871710
101	0.531883675	0.430605644	0.424598158	0.423442887
102	0.575499875	0.463498127	0.456572207	0.455390780
103	0.619920292	0.497657038	0.489791113	0.488590583
104	0.664552046	0.532897488	0.524089915	0.522878444
105	0.708716152	0.568989097	0.559262200	0.558049008

Source: the author's work

Tab. 2: Premiums on insured sum 10,000 €; x=47 old aged person; n=20 years

	Statistical Office of the SR [€]	w.r.t. $g_c(x) = cx + 1 - c$ [€]	w.r.t. $g_\gamma(x) = 1 + \gamma \cdot x^2$ [€]	w.r.t. $g_a(x) = a \cdot \left(\frac{1}{a}\right)^x$ [€]
Single premium Term Life Insurance (9)	1,629.76	1,605.75	1,605.60	1,605.77
Single premium Pure Endowment Insurance (12)	6,665.63	6,686.37	6,686.49	6,686.35
Monthly premium Term Life Insurance (8)	11.70	11.58	11.58	11.58
Monthly Premium Pure Endowment (11)	36.19	36.28	36.28	36.28

Source: the author's work

Based on Table 2, it is possible to see how the effect has the moving mixture function on the amount of individual premiums. The effect is particularly clear for single premiums in both cases. Because our approach gives a lower probability of death, we can use it as a risk loading of mortality. It is obvious, that single premium for term life insurance is smaller and for pure endowment higher. However, the obvious difference is lost in the case of monthly premiums. But, for commercial insurance companies, these differences are also very significant.

Conclusion

In life insurance, the amount of client's premium usually depends on the amount and term of his insurance and the type of policy he wants. However, the main factor that determines the premium is his age. The whole idea of adding a loading, be it medical, occupational or residential, is that life insurance will work only if the premium charged to each individual is equitable and proportionate to the risk that the life brings to the insurer. Some insurers and financial experts say that loading is justified in some cases, while in others it may not be justified.

A consumer should enquire about the reasons for loading, conditions of loading, percentage of loading, whether it is applicable for subsequent renewals and will there be any increase in the loading as the age progresses. He also needs to remember that a loading policy differs from company to company. This is because every company has its own underwriting guidelines. In most cases of life insurance, if claims have been made and the consumer decides to port to another insurer, loading will still be applicable.

Another way how insurance companies can increase profits is choosing suitable function of the risk loading to get lower mortality. These differences among functions are negligible for clients. But for the insurance company it represents a decent extra profit.

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References

1. Beliakov, G., Bustince Sola, H., Calvo Sanchez, T.: A Practical Guide to Averaging Functions. Studies in Fuzziness and Soft Computing 329. Springer International Publishing Switzerland, (2016).
2. Beliakov, G., Pradera, A., Calvo, T.: Aggregation functions: A Guide for Practitioners. Springer. Berlin, (2007).
3. Currie, I. D., Durban, M., Eilers, PHC.: Smoothing and forecasting mortality rates. Statistical Modelling, 4, 4, pp. 279-298. (2004). DOI: 10.1191/1471082X04st080oa. http://www.maths.ed.ac.uk/~mthdat25/mortality/Smoothing-and-forecasting-mortality-rates.pdf
4. Dickson, D. C. M. et al.: Actuarial Mathematics for Life Contingent Risks. New York: Cambridge University Press, (2013).
5. Euro area yield curves. https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html
6. Grabisch, M., Marichal, J. L., Mesiar, R., Pap, E.: Aggregation functions. Book Series: Encyclopedia of Mathematics and its Applications, vol. 127, Cambridge University Press, (2009).
7. Hudec, S., Špirková, J.: Mixture function as an appropriate smoothing of mortality rates. Proceedings of the International scientific conference RELIK 2017. Praha (2017).
8. Hudec, S., Špirková, J.: Smoothing of Mortality Rates Using Mixture Functions. Mortality Rates Smoothing Using Mixture Function. In: Medina J. et al. (eds) Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and

Foundations. IPMU 2018. Communications in Computer and Information Science. vol 853. Springer, Cham, pp. 140-150, (2018).
9. James, G., Witten, D., Hastie, T., Tibshirani, R.: An Introduction to Statistical Learning with Applications in R. Springer New York Heidelberg Dordrecht London, (2013).
10. Richards, S. J.: Detecting year-of-birth mortality patterns with limited data. Journal of the royal statistical society series a statistics in society, 171. Part: 1. pp. 279-298, (2008).
11. Statistical Office of the Slovak Republic. https://slovak.statistics.sk/.
12. Špirková, J.: Dissertation thesis. Weighted aggregation operators and their applications. Bratislava. 2008.
13. The Human Mortality Database. https://mortality.org/.
14. Wickham, H.: ggplot2 Elegant Graphics for Data Analysis. Second Edition. USA: Springer (2016).
15. Zimmermann, P.: Modeling mortality at old age with time-varying parameters. Mathematical Population Studies, 24:3, pp. 172-180, (2017).

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