HEALTH STATE ESTIMATION AND THE OPTIMAL RETIREMENT AGE

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Abstract

Having established a quantitative methodology of estimating the health state function of a population, we can calculate the gradual loss of health due to aging. According to the theory developed the health state is at level one at birth and gradually declines to zero at death. Note that the health state of an individual as a stochastic process (an unpredictable process during time) cannot be estimated quantitatively. Fortunately, the theory of the stochastic processes provided a quite effective tool to estimate the mean value of the health state of a large number of individuals, a population. The form of the health state function estimated for several countries is a declining curve with a negative slope by means of an accelerating decline until the end of life. This was the expected health state development due to aging. The development of the health state function provides quantitative tools for estimation of the health state of the human capital of a population and thus providing a supporting tool for the estimation of the average exit age from the labour force and estimating the optimal retirement age. Applications are presented.

Key words: Health state function, Deterioration function, health state curves, deterioration curves, retirement age.

JEL Code: I15, I18, J1

Introduction

After introducing and applying the theory of the first exit time of a stochastic process from a barrier and published it in several papers and books, we use the findings to estimate the optimal retirement age of the population. Important results are in the two books published recently in volumes 45(2017) and 46(2018) in The Springer Series on Demographic Methods and Population Analysis by Skiadas and Skiadas while other publications from the authors (2010, 2014, 2015) support the theoretical and applied material. Other related publications are due to Janssen and Skiadas (1995) and Skiadas and Zafeiris (2015), Torrance (1976), Carey et al (1992) and Weitz and Fraser (2001).

1 Related Theory

Following the first exit time theory we assume that the health state of an individual is expressed by a stochastic function denoted by S_x and the associated stochastic paths over time t or age x are estimated after integrating the stochastic differential equation

$$dS_x = h_x dx + \sigma dW_x, \tag{1}$$

with drift h_x and finding the formula for the stochastic paths S_x

$$S_x = \int_0^x h_s ds + \sigma \int_0^x dW_s = H_x + \sigma W_x, \tag{2}$$

where W_x is the Wiener process and the Health State H_x is provided as the integral of the instantaneous change h_x

$$h_x = \frac{dH_x}{dx}. \tag{3}$$

The death occurs when $S_x = 0$ and from (2) follows that

$$H_x + \sigma dW_x = 0. \tag{4}$$

The simpler form for the Health State H_x should be a decreasing process of the form (see related bibliography in Janssen and Skiadas 1995 and Skiadas and Skiadas 2007, 2010, 2013, 2014, 2015):

$$H_x = l - (bx)^c, \tag{5}$$

where l, b *and* c are parameters. The form of (4) becomes

$$l - (bx)^c + \sigma W_x = 0. \tag{6}$$

This is a very important relation providing different ways of simulation of the stochastic process.

This is demonstrated by observing the new form of (2) that is

$$S_x = l - (bx)^c + \sigma W_x. \tag{7a}$$

This form is important for constructing stochastic paths for the health state S_x.

The next form arises from (7a) by a simple transformation

$$S_x + (bx)^c = l + \sigma W_x. \tag{7b}$$

A third form given by

$$S_x - l + (bx)^c = \sigma W_x. \tag{7c}$$

The three forms (7a), (7b) and (7c) are mathematically the same. However, they provide three distinct simulation opportunities very important to explain the development of the health status and the development of the death probability density function.

According to the theory developed (Janssen and Skiadas, 1995) first it was solved the associated Fokker-Planck equation for the appropriate boundary conditions in order to find the transition probability density function and then we could find the formula for the first exit time probability density function $g(x)$ for the health state stochastic process crossing or hitting for the first time a barrier set to zero.

$$g(x) = \frac{\lfloor H_x - xH'_x \rfloor}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[H_x]^2}{2\sigma^2 x}}. \tag{8}$$

1.1 Stochastic Simulations

According to the theory developed earlier we recall equation (5) by setting $l = 1$ that is

$$H_x = 1 - (bx)^c. \tag{9}$$

This simpler form is very important for the applications and simulations that follow.

It is assumed that the health state H_x is one at zero ($H_0 = 1$) and then the health state is decreasing slowly in the first period of the life time and faster in the middle and later stages as far as the deterioration term $(bx)^c$ is getting larger and larger growing exponentially. We call as the deterioration function (*Det*) the following relation

$$Det = (bx)^c. \tag{10}$$

Now by using equation (9), equation (8) takes the form

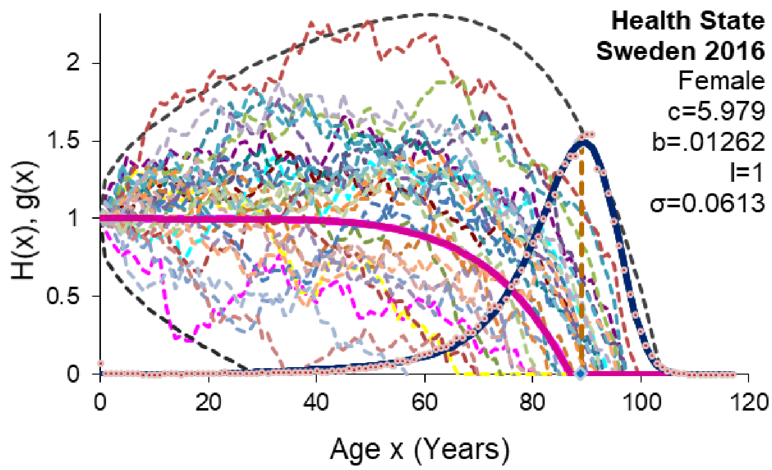
$$g(x) = \frac{1 + (c-1)(bx)^c}{\sigma\sqrt{2\pi x^3}} e^{-\frac{[1-(bx)^c]^2}{2\sigma^2 x}}. \tag{11}$$

Fig. 1: Stochastic Simulations (First method)

Fig. 1 illustrates the Health State for Sweden female in 2016. The mean health state $H(x)$ is presented by the heavy magenta curve. The health state of the individuals $S(x)$ is expressed by light dashed lines, whereas the confidence intervals are represented by the two dashed black curves. The left skewed death distribution function $g(x)$ is expressed by the yellow dashed curve with a perfect coverage with the blue fit curve. We have also reproduced the death distribution function by using the health state $H(x)$, the variance \sigma and the standard Wiener function to account for the stochastic character of the health state of the individuals

$$S(t) = H(t) + \sigma[W(t) - W(0)].$$

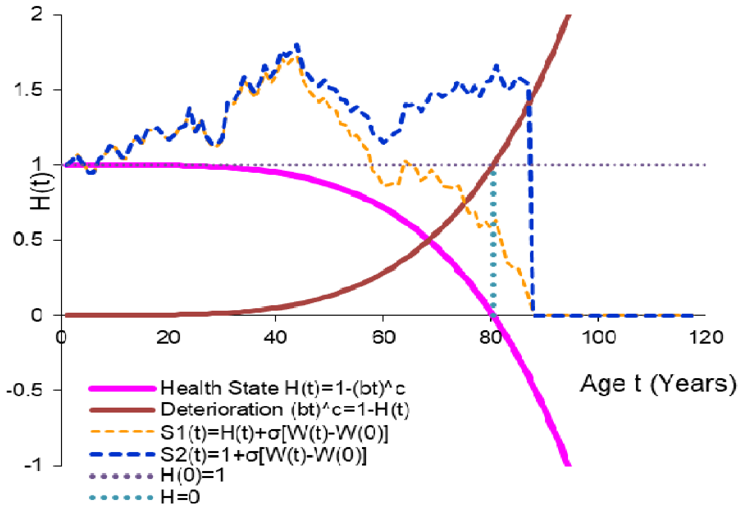
Fig.1. Health State and Stochastic paths.



Source: authors calculations

We assume that the health state of every individual starts at level 1 at birth and then follows a stochastic process at higher or lower values until the end which takes place when the stochastic path of an individual reaches the zero health level represented by the horizontal axis X at a point at age x. By collecting the number of the first exit time individual health states at every point x of the X axis, the death distribution function is derived. This is a first exit time probability density function $g(x)$. This function is found in the yearly data collected by the bureau of the census in every country. Following the theory already developed we can find the Health State of the population $H(x)$ from $g(x)$ and also reproduce $g(x)$ from $H(x)$ by stochastic simulation. This first exit time or hitting time theory was applied earlier in applications in physics when observing stochastic processes including the Brownian motion phenomena. That is completely new is the derivation of the appropriate first exit time probability density function in a form expressing the human health state problem that it to extract the health state function from the death probability density function. As we already have shown in various publications the derivation of the health state function $H(x)$ was very important for expressing the aging process from another point of view. The health state is a declining process over time whereas mortality is a growing process over time.

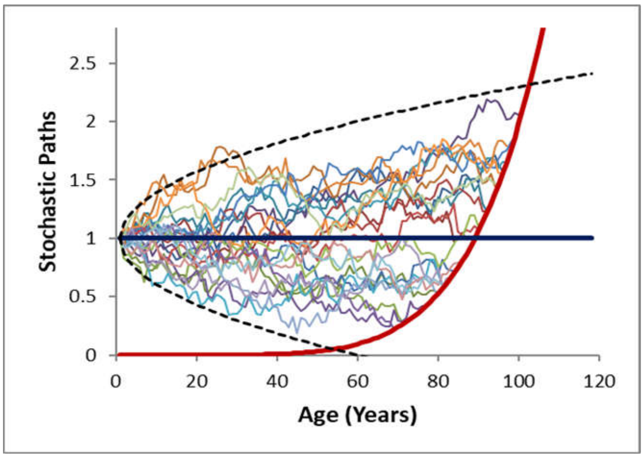
Fig.2. Health State, Deterioration and Stochastic paths.



Source: authors calculations

In figure 2 we present on how the two methods of stochastic estimation and simulation are equal. There are two main lines to simulate and reproduce the death distribution function used to estimate the health state function and the deterioration function. In the first presented in figure 1 the health state function is used as a guide of the stochastic paths with barrier the horizontal axis, whereas in the second case presented in figure 3 the deterioration function is used as a barrier to stop the stochastic paths. Both methods produce identical results as is presented in figure 2 where two stochastic paths based on the first and the second approach stop at the same age the $f1(x)$ at the X axis barrier and the $f2(x)$ at the deterioration barrier. Clearly the deterioration approach is simpler as it avoids to pass to negative values as for the health state function. The latter characterizing a critical and supercritical health range for the ages included in negative part. Note that the zero health level corresponds to ages just below the “modal” age at death (this is the age with the maximum of the death distribution).

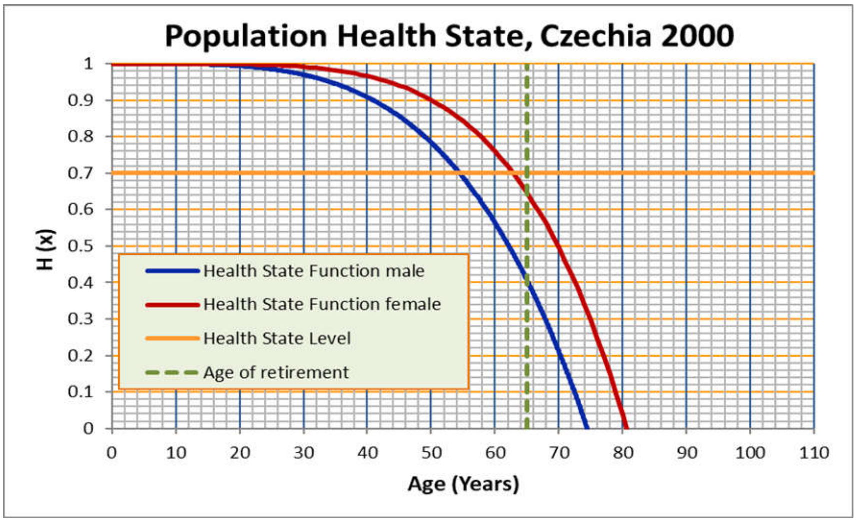
Fig. 3. Deterioration function and stochastic paths.



Source: authors calculations

The Deterioration function expressing how the mean deterioration of the population is growing is presented in figure 3. It is an exponential like function of the form $Det = (bt)^c$. In the first stages is slowly varying while it is fast growing in the last stages in the high age levels. It thus set a barrier for the health state of the individuals expressed in the graph by light lines. The two dashed curves express the confidence intervals of the stochastic paths of the individuals. The big majority of the health states (99%) are included within these intervals. The exceptional cases of people living longer are stopped by the deterioration curve.

Fig. 4. Health State in the Czech Republic for 2000.



Source: authors calculations

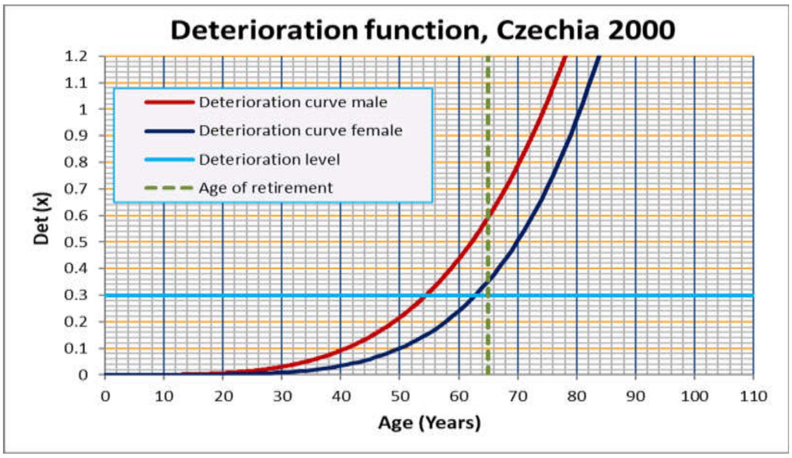
An interesting application of the health state theory is presented in figure 4 for the Czech Republic for male and female. This graph form is very useful when decide for the optimum retirement age. The very important is to keep people working with a relatively high health state. In the presented example we set a barrier at the 70% level of the total health state for the optimal retirement age. The Health state for female is higher than male. Other estimates can be found by moving the barrier in higher or lower places to account for the scenarios of decision makers.

Table I includes the years of age at 70% health state along with the differences between female-male for the various periods investigated. It is important to have as small as possible difference between sexes in the case of decisions of equal age of retirement. The difference was rather high for all three time periods with 10.5 years in 1990, dropping to 8.4 in 2000 and growing to 9.4 in 2016. In all cases the difference is quite large and needs further measures to reduce the gap.

As one of the standard barriers for retirement is 65 years of age we have set this barrier to have more information. The Health Level (%) at retirement at age 65 is presented in Table II. The health state differences are very large in this case especially for the year 1990 where the health differences between female – male are 31.6% with a clear improvement in the year 2000 at 23.9% and a small improvement at 22.8% the year 2016.

Figure 5 illustrates the deterioration function for the Czech population in 2000. Female show also higher level of survival than male. The deterioration level corresponding to 70% health state level is 0.30.

Fig. 5. Deterioration function in the Czech Republic for 2000.



Source: authors calculations

Tab. 1: Age at 70% Health level

Year	Age at Health Level 70%		Age Difference
	Female	Male	Years
1990	60.2	49.7	10.5
2000	62.8	54.4	8.4
2016	68.1	58.7	9.4

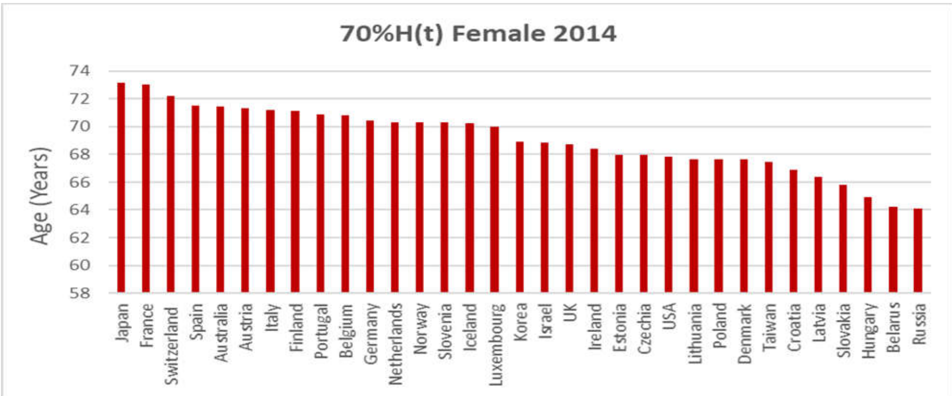
Source: authors calculations

Tab. 2: Health level (%) at retirement at age 65

Year	Health Level (%) at retirement at age 65		Health Difference (%)
	Female	Male	Years
1990	57.4	25.8	31.6
2000	64.7	40.8	23.9
2016	77.2	54.4	22.8

Source: authors calculations

Fig. 6. Age at 70% health state level for various countries

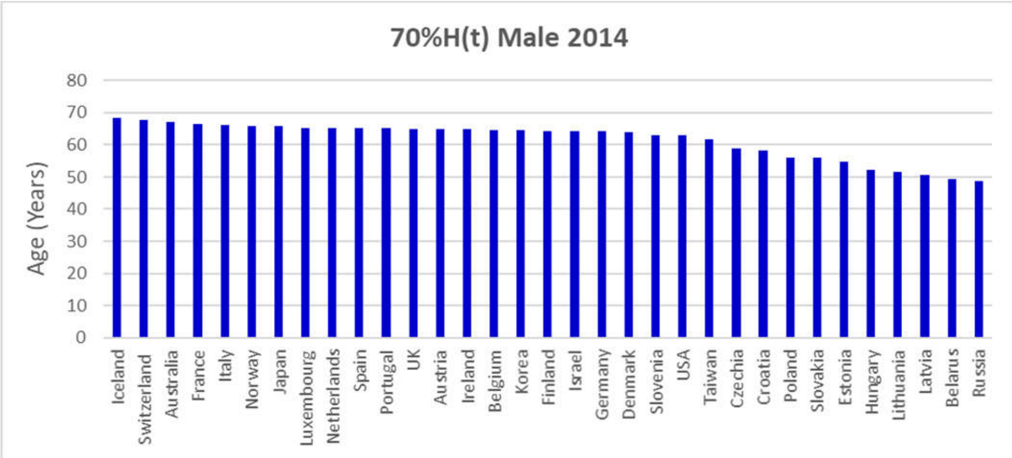


Source: authors calculations

The age at 70% health state level for female in various countries is illustrated in figure 6. Japan, France, Switzerland, Spain and Australia are at the top 5 places whereas Latvia, Slovakia, Hungary, Belarus and Russia are in the last places. The Czech Republic is ranked in the 22 place just before USA.

The age at 70% health state level for male in various countries is illustrated in figure 7. Iceland, Switzerland, Australia, France and Italy are at the top 5 places whereas Hungary, Lithuania, Latvia, Belarus and Russia are in the last places. The Czech Republic is ranked in the 24 place just before Croatia.

Fig. 7. Age at 70% health state level for various countries



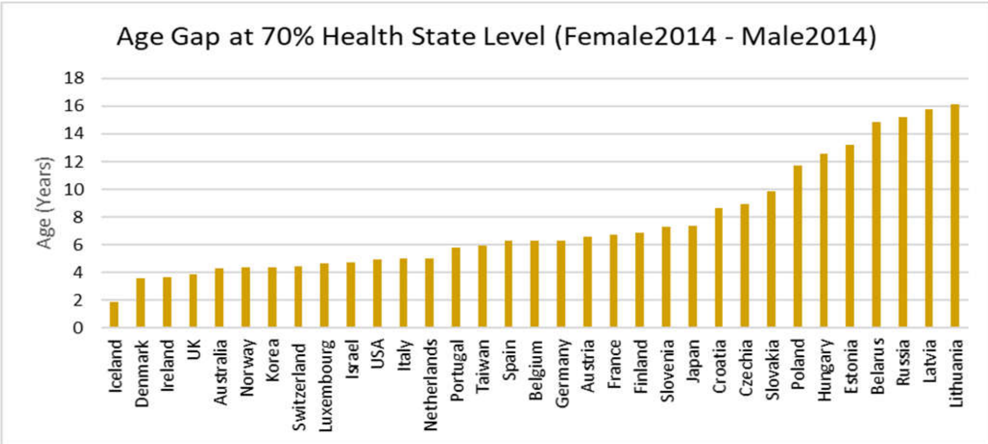
Source: authors calculations

2 Optimal Retirement Strategy

Retirement age and related decisions and applications are the main issue in governmental decisions in almost every country in the World. Especially after the huge improvements in the health systems and the advancements in life expectancy leading in an "aging society" the question is what is the optimal retirement strategy. Many talks have to do with an equal retirement age for male and female. The quest for equality is a classical approach and tends to be a leading opinion for many people especially from countries with high levels in life expectancy. However, it looks like that closing the gap between life expectancy of females and males is also a very important task. Our approach of estimating the health state of the population along with the deterioration function give important tools for assessing the health state of the population along with determining the optimal retirement strategy. The important new findings with the new method is that we have a quantitative method for estimating the health state of the population or of the deterioration function in the various age levels thus providing a simple and powerful tool when dealing with the optimal retirement strategy. Retirement has not only to do with age levels and economic decisions but mainly to succeed in giving the opportunity to people after retirement to start their new life at a relatively high level as to enjoy the remaining years of life. Another point is also to select the retirement age level after reducing the health level gap between female and male. This gap is presented in figure 8. The smallest gap with 1.8 years of age is due to Iceland. Denmark, Ireland and UK follow with the gap of over 3 years and less than 4 years. Less than 5 years gap but larger than 4 years are followed by UK, Australia, Norway, Korea, Switzerland, Luxembourg, Israel, USA and Italy. The last 7 countries in the rank are Poland, Hungary, Estonia, Belarus, Russia, Latvia and Lithuania show a gap larger than 10 years of age. The Czech Republic with 8.9 years of age gap is ranked 25 in a higher place than Slovakia and in a lower than Croatia and 2 places below Japan.

Clearly the countries with a gap higher than 5 years of age have to take strong measures to reduce the gap. This in an important action along with the equalization of the retirement age for both male and female.

Fig. 8. The Gap Between Females and Males



Source: authors calculations

Conclusion

We have applied a stochastic theory of the health state function to develop appropriate graphs useful to estimate the health state of a population or the alternative the deterioration function at several periods of time and age. The related methodology is used to find the health state at the retirement age or the retirement age at an optimum health level. The results should be useful for decision and policy makers to develop the optimal retirement strategy in a country.

References

Janssen, Jacques and Skiadas, Christos, H. Dynamic modelling of life-table data, *Applied Stochastic Models and Data Analysis*, 11, 1, 1995: 35-49.

C. H. Skiadas and C. Skiadas, A modeling approach to life table data, in *Recent Advances in Stochastic Modeling and Data Analysis*, C. H. Skiadas, Ed., World Scientific, Singapore, 2007: 350–359.

Skiadas, Charilaos and Skiadas, Christos, H. Development, Simulation and Application of First Exit Time Densities to Life Table Data, *Communications in Statistics* 39, 2010: 444-451.

Skiadas, Christos, H. and Skiadas, Charilaos. *Exploring the Health State of a Population by Dynamic Modeling Methods,* Springer, Cham, Switzerland, 2018. https://doi.org/10.1007/978-3-319-65142-2 .

Skiadas, Christos, H. and Skiadas, Charilaos. *Demography and Health Issues - Population Aging, Mortality and Data Analysis*, Springer, Cham, Switzerland, 2018. https://doi.org/10.1007/978-3-319-76002-5 .

Skiadas, Christos, H. and Skiadas, Charilaos. The First Exit Time Theory applied to Life Table Data: The Health State Function of a Population and other Characteristics, *Communications in Statistics-Theory and Methods*, 43, 2014: 1985-1600.

Skiadas, Christos, H. and Zafeiris, Konstantinos, N. Population Aging and Healthy Life: Lessons from the Related Studies, *RELIK*2015, 12-13 November 2015.

Skiadas, Christos, H. and Skiadas, Charilaos. Exploring the State of a Stochastic System via Stochastic Simulations: An Interesting Inversion Problem and the Health State Function. *Methodology and Computing in Applied Probability*, 2015, Volume 17, Issue 4, pp 973–982.

Torrance, George W. “Health Status Index Models: A Unified Mathematical View.” *Management Science*, 22(9), 1976: 990-1001.

Weitz, J.S. and Fraser, H.B. Explaining mortality rate plateaus, *Proc. Natl. Acad. Sci. USA*, 98(26), 2001: 15383-15386.

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