

# SOME METHODS FOR THE ESTIMATION OF HEALTHY LIFE EXPECTANCY

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## **Abstract**

The results of three methods developed for the calculation of healthy life expectancy will be discussed in this paper. According to the World Health Organization (WHO), the healthy life expectancy (HALE) is defined as the average number of healthy years a person is expected to live in “full health”. In order to estimate HALE, WHO has developed a very complex methodology based on the results of the Global Burden of Disease Study. An alternative and simpler approach is to apply the First Exit Time Theory on the life table death density distribution in order to calculate the healthy life expectancy under severe and moderate disabilities. In a third approach the “force of mortality”  $\mu(x)$  is used for that reason. Findings indicate that the three methods are in accordance to each other.

**Key words:** HALE, WHO, First Exit Time Theory,  $\mu(x)$ .

**JEL Code:** I15, I18, J1

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## **1 Introduction**

It was during the 1960s when the first efforts to evaluate the health status of a population took place. Sanders (1964), for example, used life table techniques in order to combine mortality and morbidity data and to measure the current health of a population. Sullivan (1971), estimated the *expectation of life free of disability*, based on abridged life table and questionnaire data for illness and disability. More recently, the World Health Organization, based on Sullivan’s method (1971) has elaborated a very complex methodology for the estimation of Healthy Adjusted Life Expectancy (HALE, see for example WHO, 2014). Also, Jansen and Skiadas (1995) and Skiadas and Skiadas (2010, 2012, 2014), have introduced the general theory of dynamics models (first exit time theory) in order to measure a Healthy Life Expectancy without severe and moderate disabilities.

The scope of this paper is to compare the results of first exit time theory with the method that is applied by WHO and to introduce a new method based on the force of mortality  $\mu(x)$ .

## 1.1 The Method used by the World Health Organization (WHO)

The method used by the WHO combines mortality data from several sources with the findings of the Global Burden of Disease Study (see Murray et al., 2012; 2015), including self-reporting data on health and disability. According to WHO<sup>1</sup> the Global Burden of Disease Study measures the “burden of disease using the disability-adjusted-life-year (DALY). This time-based measure combines years of life lost due to premature mortality and years of life lost due to time lived in states of less than full health”. However, several limitations of the method, besides its extremely high complexity (see Das and Smarasekera, 2013) and the fact that it cannot be used by any other than those who carried it out, include the lack of reliable data on mortality and morbidity for several countries. Also, there is a problem with the comparability of the self-reported data from health interviews and the measurement of health-state preferences for such self-reporting<sup>1</sup>. Thus, the uncertainty of the findings of this method must be considered as significant. This is obvious in Salomon et al. (2012) and Murray et al. (2015). In the latter which is based on the GBD Study of 2013, for the countries that will be discussed in this paper, the mean confidence intervals are on average 5.3 years in males and 6.3 years for females (see Table 2 in the results session of this paper).

In the method used by the World Health Organization the years lost due to disability (YLD) are estimated across a comprehensive set of disease and injury causes (see Vos et al., 2012, WHO, 2013). Then, the per capita fraction of YLD for all causes is calculated for every age group, sex and country, after adjusting for independent comorbidity. Based on that fraction the lost years of healthy life are calculated for each age group and the Healthy Life Expectancy at age  $x$  is the sum of healthy life years from the age  $x$  up to the open-ended interval of the life table divided by the survivors in each age  $x$  (WHO, 2013, 2014).

## 1.2 The First Exit Time Theory

Jansen and Skiadas (1995) were two of the first who applied the general theory of dynamic models to life table data in order to evaluate human health status. By definition a first exit time model for any process under consideration is described by a parent stochastic process and a boundary, which indicates a stopping condition for this process (see Ting Lee and Whitmore, 2006). In this case, the parent stochastic process is human health, which is unpredictable on a personal level. The boundary is the death, which occurs when human

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<sup>1</sup> See [http://www.who.int/topics/global\\_burden\\_of\\_disease/en/](http://www.who.int/topics/global_burden_of_disease/en/).

health falls below that boundary or barrier. Then the problem is how to find a function which describes the human health, based on this stochastic process. According to Skiadas and Skiadas (2010,2012, 2014) and Skiadas (2012a) the death density distribution  $g(x)$  or the death distribution  $d(x)$  in a life table can be modeled as:

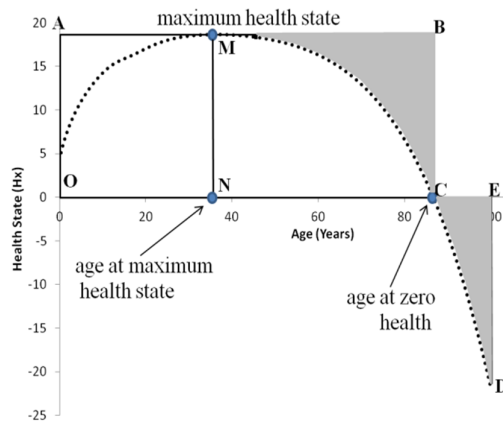
$$d(x) = k(l + (c-1)(bx)^c)(x)^{-3/2} e^{-\frac{(l-(bx)^c)^2}{2x}} \quad (1)$$

where  $x$  is the age and  $k, l, c, b$  parameters which need to be estimated; thus originally a non linear regression model is applied to the  $d(x)$  distribution of a full life table. Then it can be proved that the Health State Function  $H(x)$  can be estimated as:

$$ABS \left( -2x \ln \frac{d_{(x)} \sqrt{x^3}}{k} \right)^{\frac{1}{2}} \quad (2)$$

where  $k$  is estimated as  $k = \max(d_{(x)} \sqrt{x^3})$  (Skiadas, 2012b).

**Fig 1. The health state function of the population up to the zero health age.**



The Health State Function (HSF) has the form seen in Figure 1, in which health increases up to a point and decreases afterwards in order to be 0 in a time point of human life cycle located at the older ages (age at zero health). In fact the first part of HSF within the rectangle AMNO describes the phase of development during the human life cycle in which at a particular age the “maximum health state” of an organism is observed. This point corresponds to the maximum vitality of that organism. The white area within the rectangle MBCN represents the deterioration phase of human health until its zero point. If no-

deterioration mechanism was present, or the repairing mechanism of human body was perfect during that phase, then the health state would continue following the straight line AMB which is parallel to the x axis. This is not the case of course and that leads to the gradual disruption of human health. The problem is how to estimate the “lost life years” during the deterioration phase of the human life cycle. If  $THD_{ideal}$  is the ideal total dynamics of the population a geometric solution can be given as:

$$LHLY1 = \lambda \frac{OABC}{THD_{ideal}} \frac{THD_{ideal}}{MBCM} = \lambda \frac{OABC}{MBCM}$$

where  $\lambda$ , is a parameter expressing years and should be estimated for every case and MBCM the grey area of the rectangle MNCB. It was found that for purposes of multiple comparison of countries  $\lambda$  could be set to be 1 year.

However, the above formula has to be expanded further if the health state of the people living beyond the age at zero health is taken into consideration. In fact these people contributed to the health state of the population and for the sake of visualization they are represented in the area ECD. Then the equation above can be expanded in order for a new estimation (LHLY3) to be calculated as:

$$LHLY3 = \lambda \frac{OABC + ECD}{MBCM} \quad (3)$$

Based on the last equation the healthy life expectancy without severe and moderate disabilities (HLEB3) can be simply calculated as: (Life Expectancy at Birth)-LHLY3 (Skiadas and Skiadas, 2012).

### 1.3 The $\mu(x)$ based approach

This approach, which is presented in details in another paper in this volume by Skiadas and Zafeiris, is based on a two parameters Gompertz-like model:

$$\mu_x = \left(\frac{x}{T}\right)^b$$

where  $(x)$  is the age and  $\mu(x)$  the relevant mortality rate.  $T$  represents the age at which  $\mu(x)=1$  and  $b$  is a parameter expressing the curvature of  $\mu(x)$ . It is proven that the loss of healthy life years LHLY can be estimated as  $LHLY=\lambda(b+1)$ , where  $\lambda$  is a correction multiplier which can be set to 1 in order for different countries to be compared. Accordingly, the healthy life expectancy (HLE) is  $LEB-LHLY$ , where  $LEB$  is the life expectancy at birth.

## 2 Data and Methods

Data come from the official web page of the World Health Organization (WHO) (<http://apps.who.int/gho/data>), on the form of 5-year abridged life tables, closing at the age of 100. At the younger ages the age classes <1 year and 1-4 years are distinguished. In order for the first exit time method to be applied the available life tables were unabridged with the application of formula (1). A non-linear fitting method was applied to the original  $d(x)$  distribution, following equation (1). The unknown parameters of (1) were estimated in an EXCEL sheet with the aid of EXCEL SOLVER, in order for the sum of square errors of the fitting process to be minimized. Afterwards, the tables were unabridged with the aid of (1) for every year of life until the age of 110 years. Then the health state function was estimated according to equation (2) and the HLEB3 as described above with the aid of equation (3). The  $\mu(x)$  based method accordingly was also applied in an EXCEL sheet, in which subsequent calculations of the parameter  $b$  are done in order for the coefficient of determination of the fit to be maximized.

Data from Austria, the Czech Republic, Germany, Poland, Switzerland, Hungary, Slovakia and Slovenia, most of them countries of Central Europe, will be used in the analysis.

## 3 Results

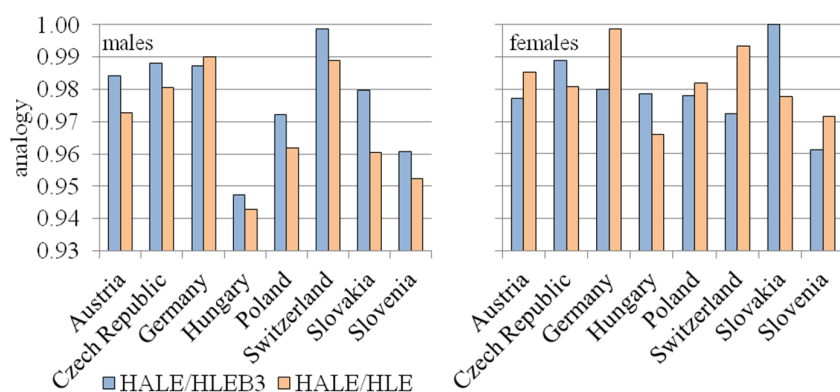
There is a significant variability among the countries of Central Europe concerning life expectancy at birth in 2013 (Table 1). In males LEB is as low as 71.4 years in Hungary and as high as 81.3 years in Switzerland, which gives a range of almost 10 years. In females, this range is lower, about 6 years between the countries of lower (Switzerland) and higher (Hungary) mortality.

**Tab. 1: Life expectancy at birth, Healthy Life expectancy (HALE) according to WHO, HLBE3 (first exit time theory) and HLE for year 2013.**

Country	LEB (WHO)		HALE (WHO)		HLEB3		HLE	
	Males	Females	Males	Females	Males	Females	Males	Females
Austria	78.7	83.1	68	73	69.1	74.7	69.9	74.1
Czech Republic	75.7	83.8	66	71	66.8	71.8	67.3	72.4
Germany	77.1	83.3	69	73	69.9	74.5	69.7	73.1
Hungary	71.4	78.8	61	68	64.4	69.5	64.7	70.4
Poland	73.1	80.6	63	71	64.8	72.6	65.5	72.3
Switzerland	81.3	84.8	71	74	71.1	76.1	71.8	74.5
Slovakia	72.4	79.9	63	70	64.3	70.0	65.6	71.6
Slovenia	77.1	83.6	66	72	68.7	74.9	69.3	74.1

According to calculations done by WHO (Table 1), healthy life expectancy (HALE) is about 9-11 years shorter than LEB in males, except of Germany where it is 8 years. If Germany is excluded, then an almost perfect linear relationship between LEB and HALE is observed ( $[HALE=(0.9316*LEB)-5.0647]$ ,  $R^2=0.98$ ). Females, whose mortality is lower, spend more time with burdened health than males: from less than 10 years in Poland up to almost 12 years in Slovenia and 13 years in the Czech Republic, despite both being of the lower mortality countries. If the Czech Republic and Slovenia are excluded, as in males, the relationship between LEB and HALE is linear ( $[HALE=(0.9509*LEB)-6.2345]$ ,  $R^2=0.96$ ).

**Fig. 2: The HALE/HLEB3 and HALE/HLE analogies in the countries studied.**



The other two methods give more optimistic estimations of healthy life expectancy (HLEB3 and HLE; Table 1). In males, the most important deviations are found in Hungary and Slovenia (HALE is about 95% and 96% of HLEB3 respectively; these proportions represent the ratio HALE/HLEB3; Figure 2). In Poland the relevant figure is 97%, in the other countries the convergence of the two measures becomes higher and in Switzerland the HLEB3 and HALE values are almost identical. Concerning the relationship of HALE with HLE, the relevant ratio (HALE/HLE) is more than 94% in Hungary, where the most important deviation is found, up to 99% in Germany and Switzerland, but it is obvious that HLEB3 seems to be a better indicator than HLE.

The opposite happens with females, in which the HALE values are more than 97% of the HLE ones. The estimated HALE is identical to HLEB3 in Slovakia. In the Czech Republic HALE levels are 99% of the estimated HLEB3. In Austria, Germany, Hungary and Poland the relevant values are 98% and in Switzerland 97%. The most important deviation is found in Slovenia (96%).

It seems then that the three methods are in accordance with each other. However, two more things need to be discussed. The first deals with the nature of  $\lambda$  of the relevant equations used for the estimation of HLEB3 and HLE, which was set to be 1 year. Even if this facilitates the comparison of different countries, a further research question deals with the nature and levels of  $\lambda$ , in order for the three methods to fully converge. An expected effect on  $\lambda$  could be that of the gender, but in any case several other agents might have played an important role too, among them the economic situation, several social and cultural parameters etc. Thus, this research question needs to be addressed in the near future. The other thing that must be taken into consideration is the degree of uncertainty in the calculations of the method applied by WHO, which enhances the difficulty for the interpretation of the results.

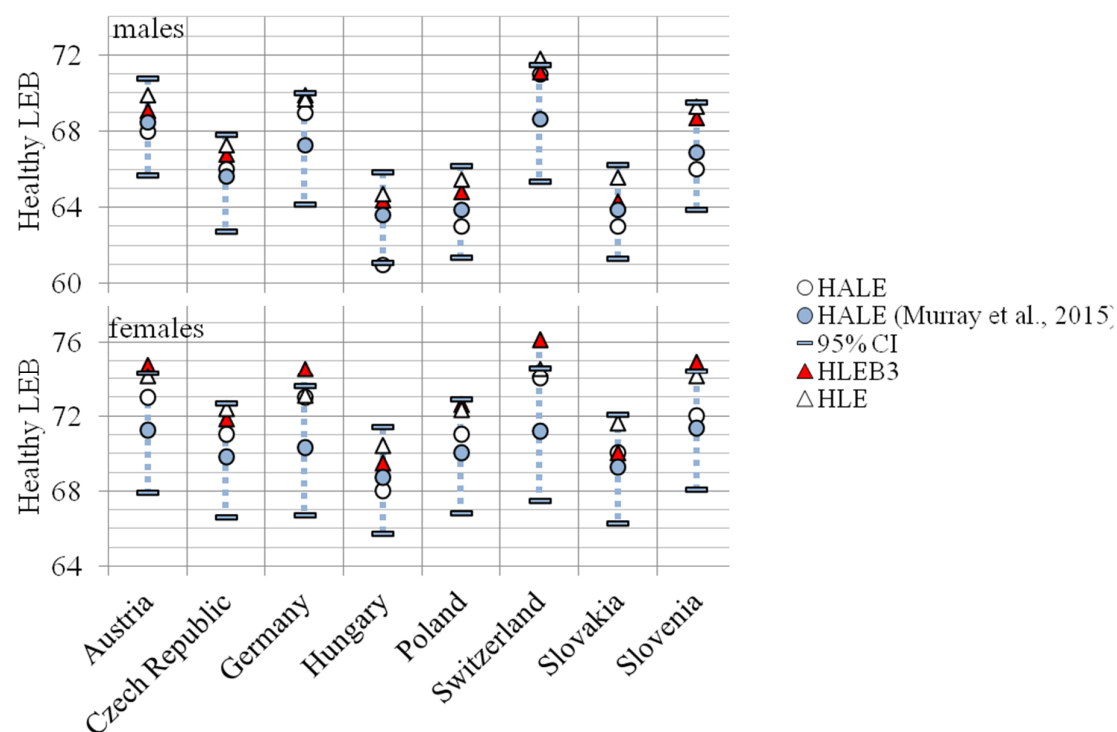
**Tab. 2: Life Expectancy at Birth, Healthy Life Expectancy (HALE) according to Murray et. al (2015) for year 2013.**

Country	Males		Females	
	LEB	HALE	LEB	HALE
Austria	78.30 (77.65-78.98)	68.47 (65.67-70.76)	83.10 (82.49-83.72)	71.21 (67.86-74.27)
Czech Republic	75.33 (74.98-75.71)	65.62 (62.75-67.85)	80.93 (80.56-81.30)	69.79 (66.58-72.65)
Germany	78.18 (77.94-78.42)	67.27 (64.18-70.02)	83.14 (82.91-83.37)	70.31 (66.67-73.60)
Poland	72.64 (71.98-73.25)	63.86 (61.35-66.20)	81.02 (80.50-81.58)	70.03 (66.77-72.88)
Switzerland	80.46 (79.81-81.09)	68.63 (65.34-71.51)	84.77 (84.17-85.35)	71.16 (67.44-74.56)
Hungary	72.18 (71.77-72.58)	63.60 (61.08-65.84)	79.26 (78.89-79.62)	68.68 (65.67-71.37)
Slovakia	72.61 (71.77-73.36)	63.87 (61.32-66.26)	79.70 (79.00-80.39)	69.25 (66.22-72.06)
Slovenia	76.86 (75.83-77.82)	66.87 (63.87-69.51)	82.95 (82.02-83.74)	71.35 (68.04-74.39)

Murray et al. (2015) gave new estimations of HALE (Table 2, Figure 3) based on the Global Burden of Disease Study of 2013. The 95% confidence intervals (CI) of these estimations are high, ranging from 5.7 years for Hungary to 6.93 for Germany in females and from 4.76 years in Hungary to 6.17 in Switzerland in males. Unfortunately WHO does not publish confidence intervals for HALE, but judging by the high degree of uncertainty denoted above, the differences found between HALE and HLEB3 seem to be not that important in males, perhaps with the exception of Hungary and Slovenia. The differences between HALE and HLE seem to be more important in Hungary, Poland, Slovakia and Slovenia. In females,

the differences of the three methods are smaller, and because the uncertainty is higher in all of the countries studied it may be hypothesized that in fact these differences are not significant.

**Fig. 3: Healthy Life expectancy at birth: HALE (WHO), HLBE3 (first exit time theory), HLE and HALE according to Murray et al. (2015) for year 2013.**



However, WHO and Murray et al. (2015) have published different results not only for HALE but also for LEB, probably because they have based their analysis in more recent data or to different analytic procedures (see methods section in the relevant publication). The differences between all the methods cited here are seen in Figure 3, where it is obvious that in almost all countries the estimations of Murray et al. (2015) are lower than those of WHO. The relevant estimations of the other methods are either within the confidence intervals of Murray et al. (2015), like for example in the Czech Republic, Poland, Slovakia in males or very close to the upper limit of confidence interval like Germany and Switzerland and Slovenia. A similar situation is observed in females.

## Conclusion

The findings of several methods developed for the calculation of Healthy Life Expectancy were cited in this paper. Findings indicate that the results of the method adopted by WHO, the  $\mu(x)$  based method and the First Exit Time Theory are in accordance with each other. Taking



into consideration the degree of uncertainty of the method used by WHO, the differences of these methods can either be considered to be not significant or may be attributed to the term  $\lambda$ , which was used in the calculations and was set to be 1 in all the countries and both genders. Further research is needed to evaluate the true levels of  $\lambda$ , in order for the results of the three methods to fully converge, keeping in mind, however, the high degree of uncertainty of the WHO method. Murray et al. (2015), probably because they are based on more recent data (Global Burden of Disease Study, 2013) or in different procedures give more “pessimistic” estimations of Healthy life expectancy than those of WHO. However, first exit time theory and the  $\mu(x)$  based approach used the data from the WHO webpage; thus in most of the cases they are in a greater accordance with the estimations of HALE done by WHO. But, additionally, it must be kept in mind a further research question concerning the differences found between HLEB3 and HLE, which cannot be discussed in this paper due to its limited size.

In any case, both the first exit time theory and the  $\mu(x)$  based approach are easier in their application, not time consuming and without any economic cost. They are based only on mortality data, which is available in most of the countries and they do not need any other data such as the results of the extremely complicated Global Burden of Disease Study. Thus, they can give estimations of the Health Status of a population in real time, facilitating the immediate implementation of policies and interventions in the health sector by governmental or other organizations.

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