MODELING OF MORTALITY: AN ALTERNATIVE METHOD OF ESTIMATING PARAMETERS OF THE GOMPERTZ–MAKEHAM FUNCTION

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Abstract

In these days there exist many discussions about lengthening of human life and about aging of population. Probably it affects structure of labor force or structure of social or health care expenditures. With the lengthening of human life is connected requirement of more accurate description of mortality at the highest ages. Mortality at these ages is affected by systematic and random errors. It is good to modify it before own calculations and use some of existing models using for modeling of mortality at the highest ages (e.g. Gompertz-Makeham function). Even if it is the oldest function, it is used very often. On the other hand, several other models exist (e.g. Kannisto and Thatcher model or polynomial functions). If we compare results for existing models, we find out that the Gompertz-Makeham function is rather pessimistic. Still it is according to some scientists (e.g. Gavrilov or Gavrilova) the most appropriate method used for modeling of mortality at the highest ages. The advantage of the Gompertz-Makeham function is that for the estimation of unknown parameters is not needed any special software. For their estimation could be used the King-Hardy methodology (which is also used by the Czech Statistical Office). The aim of the paper is to present the King-Hardy methodology and mention its advantages or disadvantages. The obtained results will be compared with these from special software. As the input data will be used data about mortality of males and females in the Czech Republic.

Key words: mortality, Gompertz-Makeham function, King-Hardy methodology

JEL Code: C02, J10, J11

Introduction

Lengthening of human life is very often discussed topic. Among very important things belongs its impact on the reforms in health and social system. Lengthening of human life also means, that number of people living at the highest ages will be higher (Arltová et al., 2013 or

Horiuchi and Wilmoth, 1998). With this it is connected requirement of the most accurate description of mortality for the oldest persons. Here is for a long time the most frequently used the Gompertz-Makeham function (Dotlačilová et al., 2014 or Šimpach, 2012).

1 Methodology

For the description of mortality is possible to use the Gompertz-Makeham function (G-M function), which belongs among the oldest ones and till this time very often used function (Boleslawski and Tabeau, 2001, Gavrilov and Gavrilova, 2011 or Thatcher et al., 1998). One of the advantage of this function is that for the estimation of unknown parameters is not necessary to have any kind of special software. We can also use formulas using for initial estimates of parameters (King-Hardy methodology) (CZSO, 2014).

Gompertz-Makeham function: $\mu_x = a + b.c^x$, (1)

where x is age, a, b and c are unknown parameters of model.

Before the estimation is good to know, that the Gompertz-Makeham function has three unknown parameters. For their estimation is important to have three equations.

In the system of equations will be used three age intervals of the same length. At first is important to determine initial age for smoothing of age-specific mortality rates and then to determine the width of age interval k (CZSO, 2014).

The first equation:

$$G_1 = \sum_{x=x_0}^{x_0+k-1} m_x = \sum_{x=x_0}^{x_0+k-1} (a + b.c^{x+0,5}) = k.a + b.c^{x_0+0,5}.(1 + c + ... + c^{k-1}), \qquad (2)$$

where m_x are age-specific mortality rates, a, b and c are parameters of the Gompertz-Makeham function.

In the same way (like in the equation G_1) we obtain the other two equations:

$$G_2 = \sum_{x=x_0+k}^{x_0+2k-1} m_x = k.a + b.c^{x_0+k+0,5}.(1 + c + ... + c^{k-1}), \qquad (3)$$

$$G_3 = \sum_{x=x_0+2k}^{x_0+3k-1} m_x = k.a + b.c^{x_0+2k+0,5}.(1 + c + ... c^{k-1}). \qquad (4)$$

If we subtract equation (3) and (4), (2) and (3) and after we divide the obtained expressions, we obtain parameter c^k:

$$c^k = \frac{G_3 - G_2}{G_2 - G_1}. \tag{6}$$

Parameter c is obtained like k-th root of c^k.

It is important to know, that the age-specific mortality rates are increasing at the highest ages and so $G_2 > G_1$. That is why it is needed to verify, if it is true:

$$b \neq 0,\ c \neq 0,\ c \neq \pm 1. \tag{7}$$

From the equation (1) is obvious, if parameter b or c will be equal to 0 or if the absolute value of c will be equal to 1 then the age-specific mortality rates will be constant (independent on the age). But it is not true at this age.

After another adjustments the initial estimate of parameter b could be obtained like:

$$b = \frac{(c-1).(G_2 - G_1)}{c^{x_0}.(c^k - 1)^2}. \tag{8}$$

Parameter a will be obtained like:

$$a = \left[G_1 - \frac{(G_2 - G_1)}{(c^k - 1)} \right] / k. \tag{9}$$

Remark 1 It is possible to perform the optimization for initial estimates of parameters of the G-M function for improvement of obtained levelling according to this function. Here it is possible to use criterion of minimization of weighted squared deviations (Fiala, 2005).

The advantage of mentioned method is its simplicity and it also gives us relatively good initial estimates of parameters. On the other hand it has some disadvantages. The obtained results are influenced by determination of initial age of smoothing (x_0) and by the width of age interval (k).

As the other method which is used for the estimation of parameters of G-M function will be used non-linear regression procedure (which use iteratively reweighted least squares). This method is implemented in software DeRaS (Burcin et al., 2012).

2 Results

For calculations and estimating of unknown parameters were used data about mortality of males and females in the Czech Republic from 1950 to 2011. Here were used the King-Hardy methodology and non-linear regression procedure, which is used for estimating of unknown parameters in software DeRaS.

Tab. 1: Estimates of parameters from MS Excel – males (Czech Republic)

CZSO - males	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
a - CZSO	0,0020883	0,0076560	0,0046798	0,0069213	0,0043086	0,0038928	0,0035135	0,0008492	0,0063900	0,0008873
b - CZSO	0,0000972	0,0000440	0,0000575	0,0000378	0,0000704	0,0000802	0,0000755	0,0001146	0,0000489	0,0001215
c - CZSO	1,0952852	1,1059909	1,1021106	1,1088420	1,1004643	1,0971962	1,0984252	1,0934537	1,1040850	1,0923391
	1960	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**
a - CZSO	0,0081812	0,0059523	0,0021042	-0,0013550	0,0006124	-0,0035365	-0,0092037	-0,0034089	-0,0056147	-0,0398019
b - CZSO	0,0000419	0,0000636	0,0001006	0,0001840	0,0001472	0,0002096	0,0003803	0,0000241	0,0002509	0,0026146
c - CZSO	1,1055892	1,1001029	1,0954858	1,0863437	1,0893631	1,0851066	1,0774181	1,1111014	1,0835344	1,0548331
	1970	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**
a - CZSO	-0,0347823	-0,0396252	-0,0188024	-0,0155799	-0,0203130	-0,0170319	-0,0214239	-0,0194567	-0,0121779	-0,0089509
b - CZSO	0,0017699	0,0021636	0,0006715	0,0004656	0,0006172	0,0004613	0,0005115	0,0004738	0,0002610	0,0002134
c - CZSO	1,0599640	1,0575333	1,0712441	1,0765535	1,0731948	1,0766043	1,0757859	1,0765137	1,0840013	1,0864726
	1980	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**
a - CZSO	-0,0018142	0,0003206	0,0075847	0,0104172	0,0171272	0,0401414	0,0238229	0,0176307	0,0088777	0,0072454
b - CZSO	0,0001039	0,0000919	0,0000372	0,0000301	0,0000111	0,0000000	0,0000037	0,0000117	0,0000504	0,0000593
c - CZSO	1,0964866	1,0973543	1,1093474	1,1120458	1,1249556	1,2228073	1,1393687	1,1229552	1,1031224	1,1012939
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**
a - CZSO	0,0071205	0,0006011	-0,0109045	-0,0426798	-0,0588809	-0,0249622	-0,0038904	-0,0017320	-0,0024318	0,0051625
b - CZSO	0,0000686	0,0001357	0,0005004	0,0031830	0,0060337	0,0011882	0,0002115	0,0001443	0,0001349	0,0000324
c - CZSO	1,0994023	1,0900598	1,0727087	1,0504630	1,0430898	1,0619293	1,0823098	1,0872311	1,0878823	1,1066903
	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
a - CZSO	0,0074212	0,0072991	0,0109698	0,0110991	0,0092936	0,0095294	0,0107796	0,0113883	0,0115183	0,0099872
b - CZSO	0,0000163	0,0000130	0,0000052	0,0000055	0,0000101	0,0000079	0,0000058	0,0000048	0,0000043	0,0000077
c - CZSO	1,1162442	1,1190259	1,1314407	1,1304659	1,1207530	1,1239270	1,1270297	1,1289739	1,1301205	1,1221086
	2010	**2011**								
a - CZSO	0,0119329	0,0080310								
b - CZSO	0,0000033	0,0000124								
c - CZSO	1,1334270	1,1149929								

(CZSO, 2014), author’s calculations

In Tab. 1 are shown estimates of unknown parameters obtained according to formulas for initial estimates (for males).

From this table it is clear, that any parameters do not infringe conditions (7). For improving of initial estimates could be used MS Excel Solver.

Tab. 2: Estimates of parameters from DeRaS – males (Czech Republic)

DeRaS - males	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
a - DeRaS	0,0028334	0,0063468	0,0061861	0,0005096	0,0017683	0,0040570	0,0009933	0,0002129	0,0055904	0,0071349
b- DeRaS	0,0000840	0,0000508	0,0000465	0,0000810	0,0001015	0,0000758	0,0000905	0,0001098	0,0000598	0,0000591
c - DeRaS	1,0964486	1,1032340	1,1041346	1,0980397	1,0948749	1,0971597	1,0954953	1,0934857	1,1005429	1,1009048
	1960	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**
a - DeRaS	0,0059650	0,0071606	0,0005022	0,0050703	-0,0001152	0,0073500	-0,0005525	0,0024168	0,0012044	-0,0104324
b- DeRaS	0,0000574	0,0000585	0,0001032	0,0000854	0,0001652	0,0000796	0,0001796	0,0001297	0,0001376	0,0005045
c - DeRaS	1,1005857	1,1001659	1,0947104	1,0955018	1,0871047	1,0962852	1,0860679	1,0904415	1,0903652	1,0741747
	1970	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**
a - DeRaS	-0,0149489	-0,0201113	-0,0048529	-0,0108651	-0,0149482	-0,0091316	-0,0186956	-0,0116982	-0,0211842	-0,0184210
b- DeRaS	0,0005457	0,0006844	0,0002382	0,0003404	0,0004025	0,0002463	0,0004349	0,0003108	0,0005305	0,0004207
c - DeRaS	1,0738046	1,0711395	1,0833734	1,0797697	1,0780307	1,0838344	1,0771151	1,0807858	1,0743371	1,0772821
	1980	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**
a - DeRaS	-0,0143769	-0,0140636	-0,0158661	-0,0086450	-0,0166931	-0,0035779	0,0049678	0,0014981	0,0058890	0,0096207
b- DeRaS	0,0003088	0,0002882	0,0003343	0,0002298	0,0003542	0,0001650	0,0000845	0,0001251	0,0000940	0,0000603
c - DeRaS	1,0817500	1,0822076	1,0803180	1,0848524	1,0794535	1,0884785	1,0967256	1,0910426	1,0941283	1,0999593
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**
a - DeRaS	0,0055716	0,0051188	0,0090976	0,0016477	-0,0006700	0,0022849	0,0040477	-0,0001066	-0,0013804	-0,0018239
b- DeRaS	0,0001010	0,0000853	0,0000580	0,0000962	0,0001198	0,0000843	0,0000703	0,0001133	0,0001095	0,0000885
c - DeRaS	1,0934836	1,0953101	1,0994283	1,0930613	1,0900433	1,0946260	1,0959597	1,0897413	1,0900954	1,0928525
	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
a - DeRaS	0,0024084	0,0032800	0,0073839	0,0056605	0,0072633	0,0065650	0,0081453	0,0080301	0,0092475	0,0128799
b- DeRaS	0,0000543	0,0000422	0,0000210	0,0000266	0,0000175	0,0000164	0,0000098	0,0000111	0,0000080	0,0000033
c - DeRaS	1,0987289	1,1015842	1,1104236	1,1076480	1,1123836	1,1131706	1,1192185	1,1169795	1,1207721	1,1322424
	2010	**2011**								
a - DeRaS	0,0115700	0,0112391								
b- DeRaS	0,0000041	0,0000051								
c - DeRaS	1,1292087	1,1259562								

(Burcin et al., 2012), author's calculations

In Tab. 2 are shown estimates of unknown parameters for the G-M function obtained by non-linear regression procedure. From obtain results it is clear, that mentioned conditions (7) are not broken. On the other hand it is clear that conditions $b>0$ and $c>1$ are realized too.

In Tab 3 are estimates of unknown parameters for the G-M function according to formulas used for initial estimates of parameters for females in the Czech Republic. From obtained results it is obvious, that the initial conditions for unknown parameters are not broken (the same as for males).

Tab. 3: Estimates of parameters from MS Excel – females (Czech Republic)

CZSO - females	**1950**	**1951**	**1952**	**1953**	**1954**	**1955**	**1956**	**1957**	**1958**	**1959**
a - CZSO	-0,0032695	-0,0017037	0,0003824	-0,0013018	-0,0032381	-0,0036462	-0,0007823	-0,0006318	-0,0040682	-0,0032026
b - CZSO	0,0000407	0,0000306	0,0000157	0,0000207	0,0000231	0,0000315	0,0000168	0,0000163	0,0000303	0,0000251
c - CZSO	1,1062092	1,1100663	1,1190441	1,1157206	1,1148740	1,1089416	1,1170187	1,1183387	1,1089421	1,1117225
	1960	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**
a - CZSO	0,0097467	-0,0009075	-0,0000979	0,0015259	-0,0002153	-0,0020110	-0,0041030	-0,0034089	-0,0023014	-0,0130147
b - CZSO	0,0000281	0,0000136	0,0000103	0,0000079	0,0000122	0,0000174	0,0000284	0,0000241	0,0000164	0,0000228
c - CZSO	1,1035955	1,1191246	1,1242545	1,1264370	1,1204879	1,1158274	1,1088080	1,1111014	1,1171000	1,1218767
	1970	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**
a - CZSO	-0,0040235	-0,0026834	-0,0002257	-0,0001232	-0,0012349	-0,0011831	0,0002462	-0,0007529	-0,0027333	0,0004884
b - CZSO	0,0000285	0,0000220	0,0000122	0,0000102	0,0000126	0,0000111	0,0000076	0,0000092	0,0000156	0,0000085
c - CZSO	1,1097618	1,1128060	1,1204030	1,1234375	1,1205994	1,1218974	1,1269918	1,1244938	1,1171735	1,1249910
	1980	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**
a - CZSO	-0,0006043	0,0014805	0,0028982	0,0025796	0,0076812	0,0078891	0,0088995	0,0059847	0,0038251	0,0009975
b - CZSO	0,0000114	0,0000069	0,0000052	0,0000057	0,0000012	0,0000012	0,0000010	0,0000021	0,0000038	0,0000077
c - CZSO	1,1219562	1,1281240	1,1318506	1,1308597	1,1514331	1,1516254	1,1540843	1,1431320	1,1347949	1,1253499
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**
a - CZSO	-0,0004016	-0,0008460	-0,0039703	-0,0056251	-0,0059006	-0,0031657	-0,0002212	-0,0002862	0,0000235	0,0022225
b - CZSO	0,0000136	0,0000144	0,0000352	0,0000485	0,0000487	0,0000260	0,0000092	0,0000098	0,0000070	0,0000027
c - CZSO	1,1171038	1,1157014	1,1029176	1,0987035	1,0985347	1,1066716	1,1202169	1,1188805	1,1232134	1,1367425
	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
a - CZSO	0,0027302	0,0036374	0,0039762	0,0044640	0,0037305	0,0043330	0,0051304	0,0046962	0,0059390	0,0042548
b - CZSO	0,0000014	0,0000007	0,0000005	0,0000005	0,0000008	0,0000005	0,0000003	0,0000004	0,0000000	0,0000005
c - CZSO	1,1455339	1,1556062	1,1612289	1,1607187	1,1521620	1,1573362	1,1623590	1,1599954	1,2052856	1,1570873
	2010	**2011**								
a - CZSO	0,0033945	0,0034018								
b - CZSO	0,0000010	0,0000009								
c - CZSO	1,14614203	1,14628925								

(CZSO, 2014), author's calculations

In Tab. 4 are results for the estimation of unknown parameters of the G-M function by non-linear regression procedure. We can conclude that also for these estimates are not broken conditions: $b>0$ and $c>1$.

Tab. 4: Estimates of parameters from DeRaS – females (Czech Republic)

DeRaS - females	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959
a - DeRaS	-0,0158543	-0,0112889	-0,0101912	-0,0077920	-0,0121500	-0,0100135	-0,0067604	-0,0084333	-0,0095181	-0,0068384
b- DeRaS	0,0001607	0,0000899	0,0000723	0,0000511	0,0000720	0,0000737	0,0000409	0,0000533	0,0000582	0,0000407
c - DeRaS	1,0877750	1,0953593	1,0979347	1,1027878	1,0989709	1,0968816	1,1042988	1,1016173	1,0996745	1,1045052
	1960	**1961**	**1962**	**1963**	**1964**	**1965**	**1966**	**1967**	**1968**	**1969**
a - DeRaS	-0,0068803	-0,0070015	-0,0071128	-0,0051109	-0,0067483	-0,0066846	-0,0094260	-0,0058893	-0,0152685	-0,0152540
b- DeRaS	0,0000420	0,0000343	0,0000301	0,0000286	0,0000396	0,0000392	0,0000527	0,0000336	0,0000451	0,0000621
c - DeRaS	1,1031634	1,1060138	1,1089936	1,1081981	1,1037713	1,1040106	1,1000579	1,1057932	1,1069488	1,1023075
	1970	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**	**1977**	**1978**	**1979**
a - DeRaS	-0,0098466	-0,0091996	-0,0080882	-0,0075854	-0,0084832	-0,0078857	-0,0052285	-0,0054779	-0,0071344	-0,0069451
b- DeRaS	0,0000581	0,0000596	0,0000448	0,0000375	0,0000408	0,0000386	0,0000242	0,0000252	0,0000363	0,0000330
c - DeRaS	1,0996847	1,0987998	1,1021825	1,1049909	1,1041013	1,1042992	1,1102761	1,1097766	1,1048914	1,1061018
	1980	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**
a - DeRaS	-0,0067642	-0,0089006	-0,0074145	-0,0075820	-0,0075779	-0,0031356	-0,0021307	-0,0003924	-0,0021720	-0,0022909
b- DeRaS	0,0000320	0,0000398	0,0000355	0,0000328	0,0000310	0,0000201	0,0000189	0,0000143	0,0000190	0,0000189
c - DeRaS	1,1073381	1,1039585	1,1052282	1,1067340	1,1071614	1,1123514	1,1131018	1,1158912	1,1119625	1,1122880
	1990	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**
a - DeRaS	-0,0007873	0,0016030	-0,0009177	-0,0007085	-0,0009294	0,0000149	-0,0027388	-0,0004665	-0,0011712	0,0001781
b- DeRaS	0,0000156	0,0000085	0,0000131	0,0000114	0,0000099	0,0000086	0,0000147	0,0000085	0,0000084	0,0000061
c - DeRaS	1,1143972	1,1218415	1,1155867	1,1174790	1,1193463	1,1210893	1,1134479	1,1203101	1,1202173	1,1243244
	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**
a - DeRaS	-0,0007642	-0,0016304	-0,0009012	-0,0002238	-0,0000277	0,0007525	0,0013556	0,0027047	0,0015902	0,0025281
b- DeRaS	0,0000064	0,0000074	0,0000058	0,0000038	0,0000037	0,0000027	0,0000021	0,0000005	0,0000008	0,0000012
c - DeRaS	1,1234405	1,1213003	1,1242877	1,1303663	1,1298401	1,1338937	1,1364397	1,1600411	1,1524806	1,1435754
	2010	**2011**								
a - DeRaS	0,0040614	0,0038196								
b- DeRaS	0,0000006	0,0000007								
c - DeRaS	1,15087937	1,14846803								

(Burcin et al., 2012), author’s calculations

Conclusion

From the obtained results it is clear, that the King-Hardy methodology and also non-linear regression procedure give us estimates not braking initial conditions. On the other hand when we use MS Excel it is also possible to do post optimization of initial estimates of parameters by MS Excel Solver.

But it is important to know, that estimates obtained from King-Hardy methodology are influenced by determination of value x_0 and by width of age interval k. This width is good to determine like that the last upper bound of age interval will be somewhere about 85 years. This age is considered as reliable from the point of values of age specific mortality rates.

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