

# PRICE SETTING OF RETAILERS UNDER THE EFFECT OF RETAIL UNIT CAPACITY AND CUSTOMER MOBILITY

**Marek Gróf - Peter Tóth**

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## **Abstract**

This paper presents a game theoretical model of price setting strategies under different consumer mobility and retail unit capacity constrictions. The theoretical model is based on a consumer grid containing four evenly distributed retailers. We study the effect of changing travel costs of consumers on the Nash equilibrium strategy of retailers, both without and with taking constricted retail unit capacity into consideration. Our findings are that under no retail unit capacity constrictions, or high retail unit capacity, higher consumer mobility directly leads to Nash equilibrium strategies with lower prices. Introducing limited retail unit capacity means that both high price and low price equilibrium strategies can exist, independent of the actual consumer mobility.

**Key words:** price setting, retail unit, consumer mobility, game theory

**JEL Code:** C70, D21

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## **Introduction**

In this paper we attempt to find solutions to a game theory representation of the retailer price setting problem, taking in to consideration both purchase and travel costs of customers, as Gärling and Gärling (1988) have shown that customers do minimize travel distances when shopping. Various models have been formulated in this field of study, ranging from the early models by Hotelling (1929) studying a linear area containing only two retailers, to more complex models using different topology. Among others, Huang and Levinson (2011) studied a market with complementary and homogenous goods, taking into account both the travel costs of customers and the actual price of goods. Here the urban area was represented by a circle comprised of the discrete locations. Granot D., Granot F. and Raviv (2010) presented a model for finding the equilibrium locations of retail outlets in an undirected weighted graph with nodes representing outlets and weighted vertices representing customers.

Our approach is to form a simple model using a grid (raster map) similar to the one used by Schelling (1969) in his studies. Further, we study the game both not including and including retailer capacity restrictions. The aim is to see, on a limited scale, what effect will different customer mobility and retailer capacity assumptions have on the existence of Nash equilibrium price setting strategies of retailers.

## 1 Model assumptions and definitions

Let  $I$  be a set of customers (individual consumers, houses, block of flats, ...) and  $J$  be a set of retailers. Variable  $d_{i,j}$  represents the travel (search) costs of customer  $i$  purchasing at retailer  $j$ , representing the mobility of customers, taking into account the distance of the customer from the retailer, and the actual price of covering the distance. Each customer purchases exactly one unit of good. All retailers have two price setting strategies  $p_j$ , where  $p_j \in \{1,2\}$ , representing the options to either sell at a low or a high price. Each customer can purchase at any retailer, with his goal being the minimization of his total purchase costs consisting of the travel costs  $d_{i,j}$  and the actual price of the purchased good  $p_j$  combined. This can be expressed as:

$$s_i = \arg \min_{j \in J} (d_{i,j} + p_j) \quad (1)$$

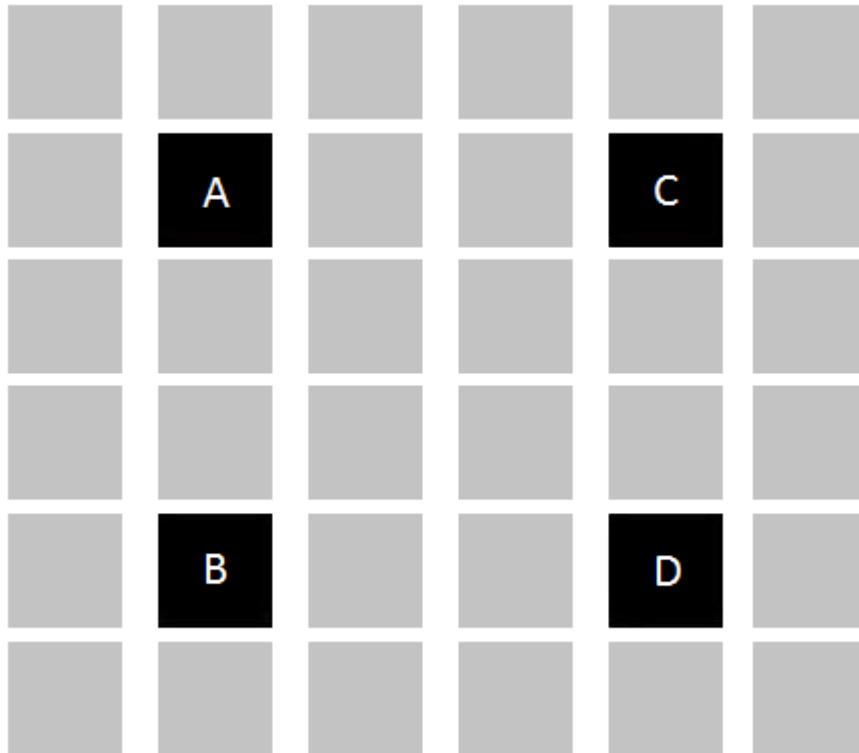
where  $s_i$  is the strategy of customer  $i$ . If the total purchase costs for a customer are the same for two or more retailers, he will split his demand between these retailers equally. The total number of customers  $D_j$  of retailer  $j$  will be equal to the number of customers that select the given retailer based on equation (1). The payoff  $M^j_{(p_j, p_{-j})}$  of retailer  $j$ , where  $p_{-j}$  is the vector of prices set by retailers other than  $j$ , can be calculated as  $M^j_{(p_j, p_{-j})} = D_j p_j$ , with retailers maximizing their payoff.

For our study we will use a theoretical area, game grid, containing 4 retailers and 32 customers, evenly distributed as shown in Figure 1. We will consider the game grid to be a torus and we will use the 8 cell Moore neighborhood, with the travel costs between any two adjacent cells to be equal to  $n$ .

## 2 Price setting under no capacity restrictions

First we will analyze the price game of retailers under no capacity restrictions and try to identify the Nash equilibrium strategies for various levels of customer mobility. First, let the travel costs be  $n = 1$ , representing normal or base mobility. In the case of high and low mobility, values  $n = 0.75$  and  $n = 1.25$  will be used, as these follow the thresholds for changes

**Fig. 1: Arrangement of retailers and customers on the game grid**



Torus game grid containing retailers (in black) and customers (in gray).

Source: own construction

in customer behavior caused by the limited number of customers in the model (a small deviation from the value of  $n = 1$  will cause no change at all unless the deviation is sufficiently large enough). Retailer payoffs under different price strategies are presented in Table 1. Note that not all possible price strategy combinations are included in the table as game grid is a torus and certain combinations are equivalent. As can be seen, under normal customer mobility the game has two Nash equilibria, with either all retailers choosing to adopt a low price strategy or all retailer adopting a high price strategy.

Second, we will study the case of high customer mobility,  $n = 0.75$ . This represents a situation when the travel costs are sufficiently low for the customers to easily react to price

changes by switching the retailer they purchase at. This leads to the game having only one Nash equilibrium strategy, where all retailers adopt a low price strategy. Any increase of price causes customers to select a different retailer, thus decreasing the payoff of the given retailer. Retailer payoffs for this case are presented in Table 2.

**Tab. 1: Retailer payoff for different price strategy combinations under normal customer mobility**

$p_j$				$M^j_{(p_j, p_{-j})}$			
A	B	C	D	A	B	C	D
1	1	1	1	8,00	8,00	8,00	8,00
2	1	1	1	6,00	10,00	10,00	9,00
2	2	1	1	8,67	8,67	11,67	11,67
2	1	1	2	6,67	12,67	12,67	6,67
2	2	2	1	12,00	10,00	10,00	16,00
2	2	2	2	16,00	16,00	16,00	16,00

Source: author's calculations

**Tab. 2: Retailer payoff for different price strategy combinations under high customer mobility**

$p_j$				$M^j_{(p_j, p_{-j})}$			
A	B	C	D	A	B	C	D
1	1	1	1	8,00	8,00	8,00	8,00
2	1	1	1	0,00	11,33	11,33	9,33
2	2	1	1	4,00	4,00	14,00	14,00
2	1	1	2	0,00	16,00	16,00	0,00
2	2	2	1	8,00	4,00	4,00	24,00
2	2	2	2	16,00	16,00	16,00	16,00

Source: author's calculations

The third possible case is the situation of low customer mobility,  $n = 1.25$ . This represents the situation where customers are not able to promptly react to price changes among the retailers, resulting in the Nash equilibrium strategy where all retailers adopt a high price setting strategy, due to the fact that increasing prices does not lead to loss of customers. The resulting payoffs are presented in Table 3.

We can conclude that, with no capacity restrictions, changes in consumer mobility directly affect the Nash equilibrium price setting strategies of retailers. In areas with low

consumer mobility the retailers are prone to adopt high price setting strategies, and in areas with high customer mobility the retailers are forced to adopt low price setting strategies due to competition with other retailers.

**Tab. 3: Retailer payoff for different price strategy combinations under low customer mobility**

$p_j$				$M^j_{(p_j, p_{-j})}$			
A	B	C	D	A	B	C	D
1	1	1	1	8,00	8,00	8,00	8,00
2	1	1	1	16,00	8,00	8,00	8,00
2	2	1	1	16,00	16,00	8,00	8,00
2	1	1	2	16,00	8,00	8,00	16,00
2	2	2	1	16,00	16,00	16,00	8,00
2	2	2	2	16,00	16,00	16,00	16,00

Source: author's calculations

We can conclude that, with no capacity restrictions, changes in consumer mobility directly affect the Nash equilibrium price setting strategies of retailers. In areas with low consumer mobility the retailers are prone to adopt high price setting strategies, and in areas with high customer mobility the retailers are forced to adopt low price setting strategies due to competition with other retailers.

### 3 Price setting under no capacity restrictions

In the previous analysis we did not take in to account the capacity of retailers, assuming they could service all the potential customers. We will now attempt to include the retailer capacity in to the model. Let  $\varepsilon = D_j / S_j$  be the retailer capacity usage, where  $S_j$  is the capacity of retailer  $j$ . We will assume that any deviation from the optimal capacity usage  $\varepsilon^*$  represents a loss for the retailer (under usage represents a situation where unused capacity is still generating costs for the retailer and over-usage of capacity represents additional costs for the retailer). As such, retailers will attempt to bring their capacity usage as close to the optimal capacity usage as possible:

$$\min \left( \frac{\varepsilon_j(p_j, p_{-j}) - \varepsilon^*}{\varepsilon^*} \right)^2 \quad (2)$$

We will now include the penalty for deviating from the optimal capacity usage in to the equation for calculating the retailer payoff, resulting in deviations from optimal capacity lowering retailer payoff:

$$M^{j,\varepsilon}_{(p_j, p_{-j})} = M^j_{(p_j, p_{-j})} \left( 1 - \sqrt{\left( \frac{\varepsilon_j(p_j, p_{-j}) - \varepsilon^*}{\varepsilon^*} \right)^2} \right) \quad (3)$$

Further, we will assume that the optimal capacity is  $\varepsilon^* = 1$ , representing 1 customer per unit of capacity and that capacity of the retailers is  $S_j = 10$  (again set in compliance with the thresholds in customer behavior due to model limitations, as in the case of customer mobility). We will again start with normal customer mobility  $n = 1$ . In this case, as can be seen in Table 4, the results are equivalent to the results with no retailer capacity, with two Nash equilibrium price setting strategies. The first Nash equilibrium strategy is for all retailers to adopt low price setting strategies, and the second is for all retailers to adopt high price setting strategies, with no possible movement between these two equilibria with no retailer cooperation.

**Tab. 4: Retailer payoff for different price strategy combinations under normal customer mobility and restricted retailer capacity**

$p_j$				$\varepsilon_j(p_j, p_{-j}) - \varepsilon^*$				$M^{j,\varepsilon}_{(p_j, p_{-j})}$			
A	B	C	D	A	B	C	D	A	B	C	D
1	1	1	1	-0,20	-0,20	-0,20	-0,20	6,40	6,40	6,40	6,40
2	1	1	1	-0,70	0,00	0,00	-0,10	1,80	10,00	10,00	6,40
2	2	1	1	-0,57	-0,57	0,17	0,17	3,73	3,73	9,69	9,69
2	1	1	2	-0,67	0,27	0,27	-0,67	2,20	9,25	9,25	2,20
2	2	2	1	-0,40	-0,50	-0,50	0,60	7,20	5,00	5,00	6,40
2	2	2	2	-0,20	-0,20	-0,20	-0,20	12,80	12,80	12,80	12,80

Source: author's calculations

The second case will be, as previously, the situation with high customer mobility  $n = 0.75$ . In contrast to the case with no restrictions imposed on retailer capacity, in this case we can see that two equilibrium price strategy combinations exist. The first one, with retailers adopting low price setting strategy, is the same as in the case with no retailer capacity restrictions. The second equilibrium strategy combination, with retailers adopting high price setting strategies, is a result of deviations from optimal capacity usage penalizing the payoff of the retailer that would adopt a low price setting strategy and attract a higher number of customers than he would be able to service. The possible price setting strategy combinations are presented in Table 5.

**Tab. 5: Retailer payoff for different price strategy combinations under high customer mobility and restricted retailer capacity**

$p_j$				$\varepsilon_j(p_j, p_{-j}) - \varepsilon^*$				$M^{j,\varepsilon}(p_j, p_{-j})$			
A	B	C	D	A	B	C	D	A	B	C	D
1	1	1	1	-0,20	-0,20	-0,20	-0,20	6,40	6,40	6,40	6,40
2	1	1	1	-1,00	0,13	0,13	-0,07	0,00	9,86	9,86	8,67
2	2	1	1	-0,80	-0,80	0,40	0,40	0,80	0,80	8,40	8,40
2	1	1	2	-1,00	0,60	0,60	-1,00	0,00	6,40	6,40	0,00
2	2	2	1	-0,60	-0,80	-0,80	1,40	3,20	0,80	0,80	-9,60
2	2	2	2	-0,20	-0,20	-0,20	-0,20	12,80	12,80	12,80	12,80

Source: author's calculations

The final situation that we will study is low customer mobility  $n = 1.25$  combined with restricted retailer capacity. In this case the retailer is not able to affect customer behavior by adopting a different price setting strategy, and as such the capacity deviation plays on role but rather only globally reduces retailer payoffs. As in the case of no retailer capacity restriction, one Nash equilibrium price setting strategy combination will exist, with all retailers adopting a high price setting strategy. The payoffs for various price setting strategy combinations are presented in Table 6.

**Tab. 6: Retailer payoff for different price strategy combinations under low customer mobility and restricted retailer capacity**

$p_j$				$\varepsilon_j(p_j, p_{-j}) - \varepsilon^*$				$M^{j,\varepsilon}(p_j, p_{-j})$			
A	B	C	D	A	B	C	D	A	B	C	D

1	1	1	1	0,20	0,20	0,20	0,20	6,40	6,40	6,40	6,40
2	1	1	1	0,20	0,20	0,20	0,20	12,80	6,40	6,40	6,40
2	2	1	1	0,20	0,20	0,20	0,20	12,80	12,80	6,40	6,40
2	1	1	2	0,20	0,20	0,20	0,20	12,80	6,40	6,40	12,80
2	2	2	1	0,20	0,20	0,20	0,20	12,80	12,80	12,80	6,40
2	2	2	2	0,20	0,20	0,20	0,20	12,80	12,80	12,80	12,80

Source: author's calculations

These cases were all studied with the retailer capacity of  $S_j = 10$ . Changes in retailer capacity would again force changes in the price setting strategies of retailers if the changes in capacity were sufficiently large enough. The number of possible mobility and capacity combinations is relatively high and will not be therefore discussed in this paper.

## Conclusion

Using a game theoretical model of customer preferences under different price setting strategies of retailers we have shown that changes in customer mobility affect Nash equilibrium strategy combinations of retailers. Under normal customer mobility both a market wide low price setting strategy and a market wide high price setting strategy are stable. Reducing customer mobility, which can be caused by lower retailer density (longer travel distances) or travel costs increasing per unit of distance traveled, leads to retailers becoming a local monopoly being able to adopt a high price setting strategy with no change of behavior from customers, leading to the market wide high price setting strategy being stable. In the opposite case of increasing customer mobility, we observed that only the low price setting strategy is stable, as any change in prices would drive customers to a different retailer. Introducing retailer capacity restriction in the form of penalizing deviations from the optimal capacity usage leads to a change only in the case of low customer mobility, where insufficient capacity causes the market wide high price setting strategy to become stable, leading to the same state as with normal customer mobility.

## References



1. D. Granot, F. Granot, T. Raviv. On competitive sequential location in a network with a decreasing demand intensity. *European Journal of Operational Research*, Elsevier 205(2): 301-312, 2010.
2. T. Gärling, E. Gärling. Distance minimization in downtown pedestrian shopping. *Environment and Planning A* 20(4): 547-554, 1988.
3. H. Hotelling. Stability in Competition. *Economic Journal*, 39: 41-57, 1929.
4. Huang, D. Levinson. Why retailers cluster: an agent model of location choice on supply chains. *Environment and Planning B: Planning and Design* 38: 82-94, 2011.
5. T. Schelling. Models of segregation. *The American Economic Review*, 59 (2), 488-493, 1969.

### **Contact**

Marek Gróf

Faculty of Economics, Technical University of Košice,

Němcovej 32, 040 01 Košice, Slovakia

marek.grof@tuke.sk

Peter Tóth

Faculty of Economics, Technical University of Košice,

Němcovej 32, 040 01 Košice, Slovakia

peter.toth22@gmail.com