DEMOGRAPHIC AND HUMAN DEVELOPMENT INDICATORS

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Abstract

We develop and apply the first exit time or hitting time theory to death and population data in 35 countries. The health state or health status is modelled and estimated for these countries and for long time periods. The results include the classical life expectancy, the healthy life expectancy, the loss of healthy life years, the expected healthy age and the total health state. Furthermore the human development age groups are estimated following the health state methodology. The findings are important to demographers, health policy makers, economists, social scientists and psychologists, gerontologists, statisticians, applied mathematicians and actuaries and insurance people. The theoretical part is published in several journals and books starting from the first publication in 1995 while many applications are done (see at: www.cmsim.net).

Key words: Health state, health status, hitting time theory

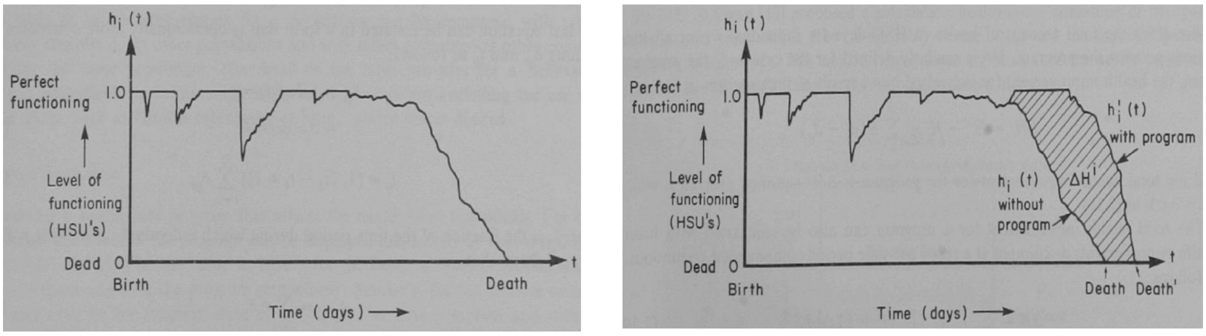
JEL Code: I15, I18, J1

Introduction

The Health State or Health Status of a Population is a crucial economic parameter important for the development and progress of a country. The measurement and estimation of health status was an important task for centuries after the rising of organized economies. However, the main serious efforts started during sixties after the vast health improvements in the late forties and fifties, due to the introduction of special medications and treatments. A relatively well organized health system and the related state health agencies make possible the systematic collection of reliable data for the health status of the population via the introduction of a Health Index. Several attempts started during sixties with main references from Sanders (1964), Chiang (1965) and Sullivan (1966). Sullivan (1971) and Torrance

(1976) continued their efforts in quantifying the Health Status process. (Torrance 1976) introduced the term *Hx* for the Health Status of the Population.

Fig. 1A: Health Profile of Individual *i* and Fig. 1B. Program Effect on Individual *i*.



Source: George W. Torrance. "Health Status Index Models: A Unified Mathematical View", Management Science, 22(9), 1976: 990-1001.

Torrance suggested a time (age) development scheme for the health status $h_i(t)$ of an individual i presented in figure 1A and how this health status could developed following a successful "program". He concluded in providing a method for estimating the health status of the population $H(t)$ of m individuals by averaging all the health status $h_i(t)$ as is presented in equation (1).

$$H(t) = (1/m)\sum_{i=1}^{m} h_i(t) \qquad (1)$$

Sullivan proposed and applied a technique using both the death and population estimates via life tables and simultaneously the information collected from questionnaires. His method is used in many applications for estimating the healthy life expectancy and the loss of healthy life years.

1 Modeling Health State

The main idea of researchers accepting and applying the Torrance methodology was to hit directly the problem by collecting health status data from a carefully selected sample of individuals and then estimating health status characteristics as is the loss of healthy life years and the healthy life expectancy of the total population. Although the estimation of health status $H(t)$ is hard to be measured in terms of Torrance a method leading in the estimation of

the loss of healthy life years and then the healthy life expectancy was proposed by Sullivan and widely used in nowadays.
Our approach (Janssen and Skiadas, 1995) was to use the findings of an advanced first exit time stochastic theory technique to estimate the health status $H(t)$ of a population from the outcome that is from the distribution of the deaths by year of age taking into account the population for every age interval that is usually 1 year in the complete life tables but also 5 years for abridged life tables. We have thus succeeded to have an estimate of the unknown but very important health state or health status function $H(t)$.

Such a process can be modeled by a simple stochastic differential equation of the form

$$dh_t = \mu_t^* dt + \sigma_t dW_t \tag{2}$$

Where h_t is the health state or the vitality of the individual, μ_t^* is a function of the age t and σ_t is the diffusion coefficient. In order to avoid confusion with the force of mortality denoted by μ in the actuarial science we have set the μ^* for the different function expressing the loss of vitality or the rate of decrease of the health state.

If we assume that μ_t^* and σ_t are independent from h_t the solution of (2) is immediate by integration.

$$h_t = H_t + \int_0^t \sigma_s dW_s \tag{3}$$

where the health state function is:

$$H_t = \int_0^t \mu_s^* ds\,. \tag{4}$$

The main problem here is not to find the solution of (2) but the transition probability density function $p(t)$. From (2) we can pass to the associated Fokker-Planck partial differential equation:

$$\frac{\partial p(h_t,t)}{\partial t} = -\mu_t^* \frac{\partial p(h_t,t)}{\partial h_t} + \frac{\sigma_t^2}{2}\frac{\partial^2 p(h_t,t)}{\partial h_t^2} \tag{5}$$

The solution is given by Janssen and Skiadas (1995) and is of the form

$$p(t)=\frac{1}{\left[2\pi\int_0^t\sigma_s^2\,ds\right]^{1/2}}e^{-\frac{(H_t)^2}{2\int_0^t\sigma_s^2\,ds}} \quad . \tag{6}$$

Then, for constant σ, the first exit time probability density function $g(t)$ is given by:

$$g(t)=\frac{|H_t-tH_t'|}{t}p(t)=\frac{|H_t-tH_t'|}{\sigma\sqrt{2\pi t^3}}e^{-\frac{(H_t)^2}{2\sigma^2 t}} \tag{7}$$

(7) cannot be solved for the unknown state function $H(t)$ given $g(t)$ (the death probability). However, by adding a correction term f_t we can find an approximation of the form (k is a constant and σ=1):

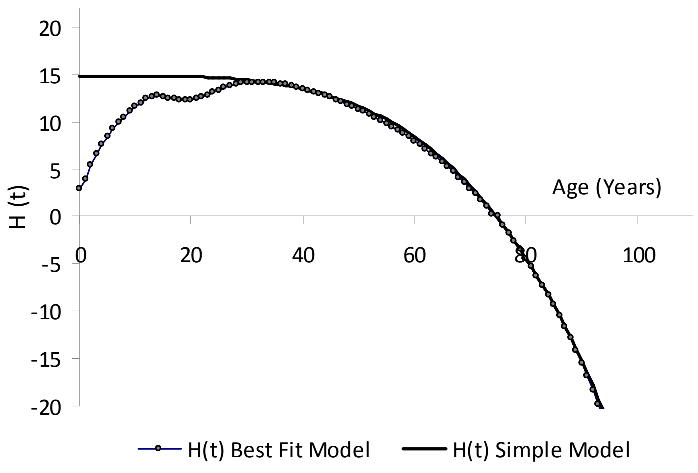
$$g(t)=\frac{k}{\sqrt{2\pi t^3}}e^{-\frac{(f_t+H_t)^2}{2t}} \tag{8}$$

Now, inversion of (8) yields immediately the following form

$$H_t+f_t=\pm\left(-2t\ln\frac{g(t)\sqrt{t^3}}{k}\right)^{1/2} \tag{9}$$

The estimation of $H(t)$ from the last formula is presented in Skiadas and Skiadas (2013)

Fig. 2: Health State for males in Czech Republic (2000)



2 Applications

Applying the theory and results from our paper, Janssen and Skiadas (1995), the health state of the Czech Republic (males, 2000) is presented by the dotted curve in Figure 2, whereas the continuous curve is in accordance to the Torrance proposals. The related to Torrance first exit time theory was proposed by Skiadas and Skiadas (2010). This simple model ($H(t)=a-(bt)^c$, a, b and c are parameters) covers a part of the real health state presented by the best fit model starting from ages next to 30 years and higher. The first part of the health state covering ages from infancy and early childhood to adolescence and early and middle ages of adult development start from earlier values than the plateau and then increases to maturity passing from a slow-down in the years aroud 20's. That is extremely important is that we have a tool and method to estimate the unknown health state curve of a population by defining the appropriate health state function.

The theory is applied to the Czech Republic life table data (males and females) from 1950 to 2005. The results are illustrated in Tables 1 and 2.

Table 1 includes the loss of healthy life years from severe (LHLY1), moderate & severe (LHLY3), light (LHLY light) and total disability causes (LHLY total). Accordingly the healthy life expectancy at birth (HLEB total)) without LHLY total, the healthy life expectancy at birth (HLEB moderate and severe) without LHLY3 and the healthy life expectancy at birth (HLEB severe) without LHLY1. Furthermore the healthy life expectancy at birth estimated from data and from fit results is included along with the expected healthy age. Finally the total health state from data and from fit is estimated. The total health state is estimated by calculating the total area under the health state curve and the horizontal axis. A clear improvement appear in all the critical indicators in the time interval studied (1950 to 2005).

Table 2 includes the estimates of specific age points of human development. These points are the maximum or minimum points for $H(t)$ and the corresponding first, second and third order derivatives. Infancy and early childhood human development is followed by the pre-adolescence period. Early, middle and late adolescence periods are followed by early, middle and late adult development periods until the top of the health stage. Then the early, middle and old age periods follow and end with the very old age period.

Tab. 1: Health State estimates for Czech Republic population

Czech Republic (Males 1950-2005)													
Healthy Life Expectancy at Birth (HLEB) & Loss of Healthy Life Years (LHLY)										Expected Healthy Age		Total Health State	
	LHLY1	LHLY3	LHLY Light	LHLY Total	HLEB Total	HLEB Moderate & Severe	HLEB Severe	LEB from Data	LEB from Fit	From Data	From Fit	From Data	From Fit
1950	5.7	9.4	3.4	12.8	48.9	52.3	56.0	61.7	61.5	34.8	35.1	884	883
1960	5.9	9.1	2.9	12.0	55.1	58.0	61.2	67.1	67.3	34.9	34.9	932	927
1970	6.2	10.0	1.4	11.4	54.8	56.2	60.0	66.2	66.1	34.2	34.5	905	904
1980	6.0	9.8	2.8	12.6	54.5	57.3	61.2	67.1	67.3	34.5	34.5	930	926
1990	5.4	8.1	3.6	11.8	56.3	59.9	62.7	68.0	68.4	34.5	34.3	928	921
2000	6.0	8.5	5.1	13.6	57.9	63.0	65.5	71.5	71.9	36.5	35.9	1036	1009
2005	6.0	8.3	6.4	14.7	58.2	64.5	66.8	72.8	73.1	37.2	36.7	1072	1048
Czech Republic (Females 1950-2005)													
1950	6.4	10.2	4.7	14.8	51.8	56.4	60.2	66.6	66.4	37.1	37.2	1021	1018
1960	6.8	10.2	5.6	15.8	57.1	62.7	66.1	72.9	73.2	37.7	37.8	1120	1120
1970	6.9	10.0	5.5	15.5	57.7	63.2	66.3	73.2	73.4	37.7	37.8	1126	1127
1980	6.5	9.3	7.3	16.6	57.7	65.0	67.8	74.2	74.5	38.0	38.0	1154	1149
1990	6.7	9.2	7.6	16.8	58.8	66.3	68.9	75.6	75.8	38.5	38.4	1185	1174
2000	6.7	9.0	8.5	17.5	60.8	69.3	71.6	78.2	78.7	39.1	39.6	1240	1250
2005	7.3	9.4	9.8	19.1	60.2	69.9	72.0	79.3	79.6	40.4	40.2	1313	1290

Source: Our estimates

Tab. 2: Human Development Stages for Czech Republic population

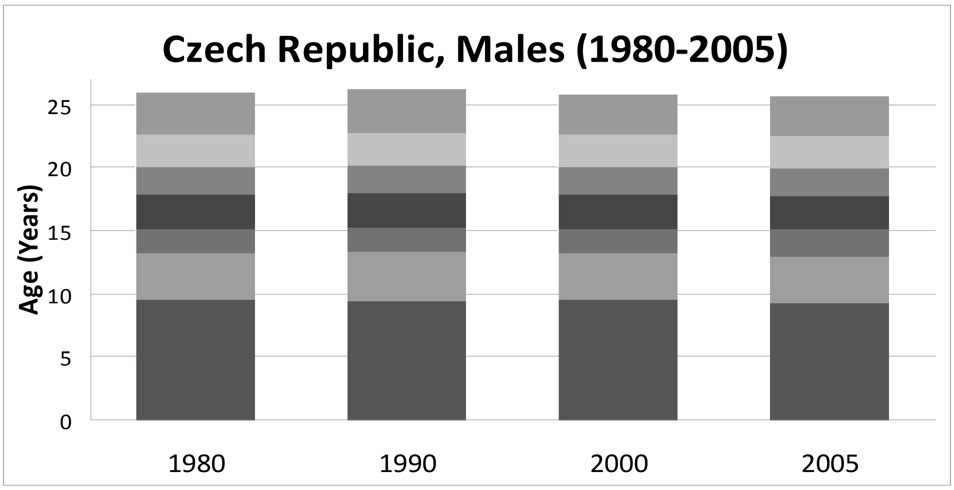
Czech Republic (Males 1950-2005)																				
Human Development Stages																				
Age Groups estimated with the Health State Methodology (Years)																				
		INFANCY AND EARLY CHILDHOOD		PRE ADOLESCENCE		EARLY ADOLESCENCE		MIDDLE ADOLESCENCE		LATE ADOLESCENCE		EARLY STAGE OF ADULT DEVELOPMENT		MIDDLE STAGE OF ADULT DEVELOPMENT		LATE STAGE OF ADULT DEVELOPMENT		EARLY & MIDDLE OLD AGES		VERY OLD AGES
1950	0		8.5		12.4		15.3		19.0		22.1		25.8		30.4		34.3		76.2	
1960	0		8.9		13.2		15.8		19.0		21.8		25.0		29.0		34.6		76.0	
1970	0		8.2		12.6		15.8		19.8		23.1		27.1		32.1		35.1		74.8	
1980	0		9.5		13.2		15.2		17.8		20.1		22.6		25.9		31.6		75.8	
1990	0		9.4		13.3		15.3		18.0		20.2		22.8		26.2		31.4		76.1	
2000	0		9.5		13.2		15.2		17.8		20.1		22.6		25.8		32.1		79.4	
2005	0		9.3		13.0		15.1		17.7		19.9		22.5		25.7		32.1		81.0	
Czech Republic (Females 1950-2005)																				
		INFANCY AND EARLY CHILDHOOD		PRE ADOLESCENCE		EARLY ADOLESCENCE		MIDDLE ADOLESCENCE		LATE ADOLESCENCE		EARLY STAGE OF ADULT DEVELOPMENT		MIDDLE STAGE OF ADULT DEVELOPMENT		LATE STAGE OF ADULT DEVELOPMENT		EARLY & MIDDLE OLD AGES		VERY OLD AGES
1950	0		9.7		12.3		16.2		21.0		25.1		30.0		36.0		38.6		79.0	
1960	0		10.3		11.5		14.3		18.0		21.1		24.7		29.2		37.4		80.5	
1970	0		9.1		12.5		14.7		17.4		19.8		22.4		25.9		37.0		80.6	
1980	0		10.5		13.2		14.6		16.5		18.1		19.8		22.2		36.7		81.1	
1990	0		10.8		13.6		14.9		16.6		18.1		19.8		21.9		35.9		82.4	
2000	0		9.7		12.7		14.3		16.4		18.1		20.1		22.7		38.4		84.2	
2005	0		9.2		11.9		13.5		15.7		17.5		19.6		22.3		39.6		85.2	

Source: Our estimates

According to our findings for Czech Republic (1950-2005, males and females) presented in Table 2, the infancy and early childhood stage corresponds to ages from 0 to ages

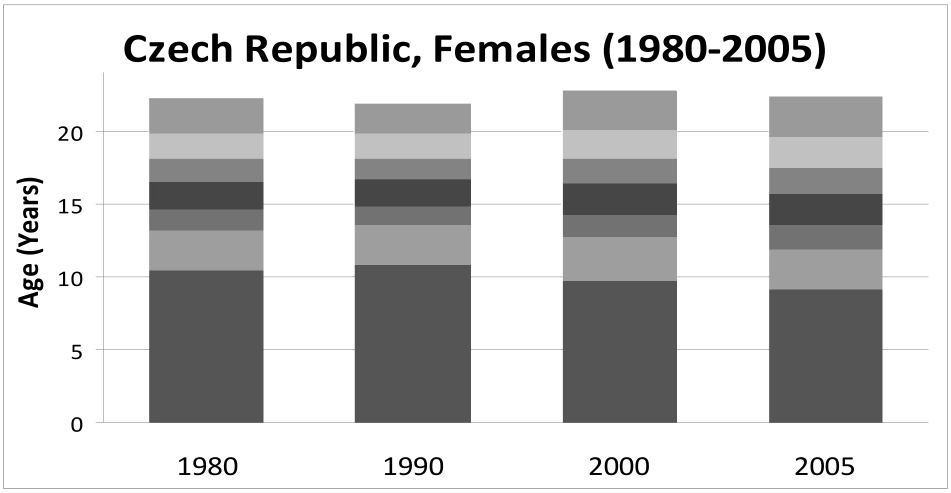
8.5 minimum and 9.5 maximum for males and 9.1 minimum and 10.8 maximum for females. No significant differences appear. Instead the pre-adolescence period tends to end at 11.9 years for females 1.1 years earlier than males at 2005. This difference was 0.1, 1.7, 0.1, 0, -0.3 and 0.5 in 1950, 1960, 1970, 1980, 1990 and 2000 respectively. The early adolescence period, with the exception of 1950 results, ends 1.5, 1.1, 0.6, 0.4, 0.5 and 1.6 years earlier for females than males for the years 1960, 1970, 1980, 1990, 2000 and 2005 respectively. The middle adolescence period, with the exception of 1950 results, ends 1.0, 2.8, 1.3, 1.4, 1.4 and 2.0 years earlier for females than males for the years 1960, 1970, 1980, 1990, 2000 and 2005 respectively. These differences tend to be close to 2 years for the late adolescence period from the years 1980 (2.0), 1990 (2.1), 2000 (2.0) and 2005 (2.4). The end of the late adolescence period is 17.5 for females and 19.9 for males in 2005. After this stage the early stage of adult development ends at 19.6 years for females and at 22.5 years for males a clear difference of 2.9 years of age for 2005. The end of middle stage of adult development tends to be close to 22.5 years of age for females and close to 26 years for males the period from 1980 to 2005. Instead the end of the late stage for adult development was 39.6 years for females and 32.1 for males in 2005. The difference between females and males is 4.3 (1950), 2.8 (1960), 1.9 (1970), 5.1 (1980), 3.5 (1990), 6.3 (2000) and 7.5 (2005). The age year of the end of the early, middle and old ages is considerably higher for females than males, 2.8 (1950), 4.5 (1960), 5.8 (1970), 5.3 (1980), 6.3 (1990), 4.8 (2000) and 4.2 (2005) years of age.

Fig. 3: Young age groups for males in Czech Republic



Source: Our estimates

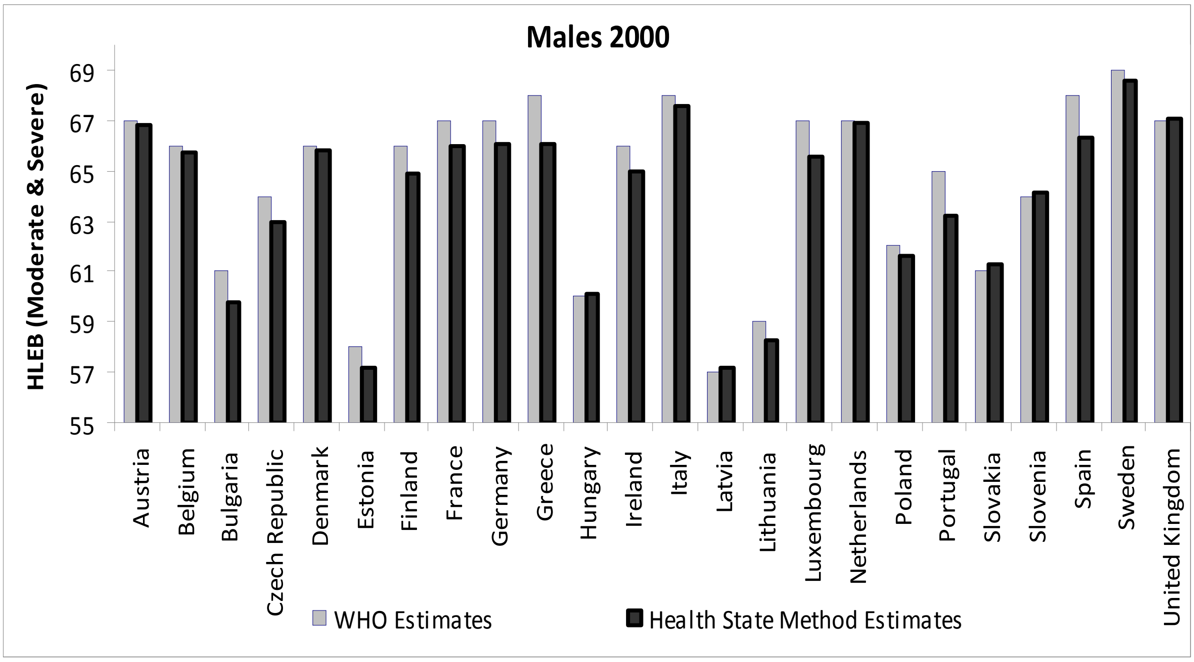
Fig. 4: Young age groups for females in Czech Republic



Source: Our estimates

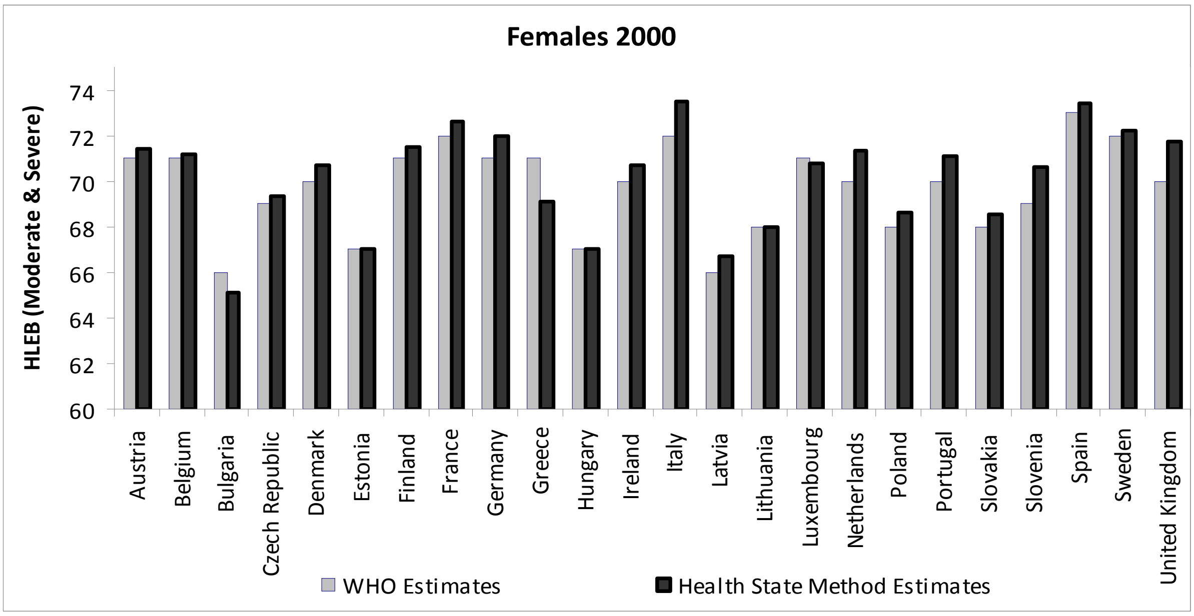
Note that, as our estimates are based on the health state curve and the related derivatives, the results based on the higher order derivatives are more sensitive especially for the period 1950 to 1970. So only the main estimates for 1980-2005 are illustrated in Figures 3 and 4. For males the main age groups are almost stable for this time period. One group covers the first 10 years of age, two groups correspond to 10-15 years (preadolescence and early adolescence), two other for 15-20 years of age (middle adolescence and late adolescence) and another two for the years over 20 (first stage of adult development and the onset to the middle stage for adult development). For females there are 5 age groups from 10 to 20 years of age and only one group, the middle age of adult development, is over 20 years of age. Furthermore, it is a clear shift of the female groups to younger ages. Preadolescence and early adolescence, middle adolescence and late adolescence and the onset to the first stage of adult development are developed in 10.4 years for females and in 13.2 years for males in 2005.

Fig. 5: Healthy life expectancy at birth without moderate & severe disability (Males)



Source: Our estimates and HALE results from WHO

Fig. 6: Healthy life expectancy at birth without moderate & severe disability (Females)



Source: Our estimates and HALE results from WHO

In Figures 5 and 6 we compare our estimates for the healthy life expectancy at birth without moderate and severe disability causes HLEB (moderate & severe) with the related estimates of the World Health Organization (WHO) termed as HALE. Both methods‘ results are very close each other. The mean absolute error is 1.1% for males and 1.0% for females. Of course, with our proposed and applied models and method, is possible to find important health and demographic parameters for all the time periods from the time when census data exist. The Human Mortality Database includes systematically collected data from last centuries with the oldest from Sweden (1751).

Conclusion

We have presented and applied the general theory of the health state of a population in several countries and comparative applications are done. The Health State Function is developed and estimated. An analysis of the Czech Republic data series from 1950 to 2005 is given and a comparative study between our results and the corresponding from the World Health Organization is presented. Both methods provide similar results while our method, not based on questionnaires, can be applied to all life table data provided from the bureau of the census by means that we can find the health status of populations far to the past as long as death and population data are provided. The demographic and human development indicators for 35 countries can be downloaded at www.cmsim.net .

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