RELIK 2013.
Reprodukce lidského kapitálu – vzájemné vazby a souvislosti. 9. – 10. prosince 2013

# DEVELOPMENT OF THE LIFE EXPECTANCY AND NORMAL LENGTH OF LIFE IN SELECTED EUROPEAN COUNTRIES

**Petra Dotlačilová – Jitka Langhamrová**

**Abstract**

From current researches it is obvious that the human life has constantly extended. One of the reasons for this positive development is better medical care the other could be higher people's interest in a healthy lifestyle. However, it is important to know that the enforcement of a healthy lifestyle is possible because of better availability of an organic food. The higher level of medical care is then possible by technological progresses.

The results of researches indicate that there is and will be an extension of human life. But how much longer can human life extend? Is there a limit which can't be exceeded? Scientists stated that the human body can live up to 120 years.

For analyzing of the development of mortality is the most commonly used an indicator known as the life expectancy at exact age *x*. But it is not the only one. As another could be mentioned modal age at death or probable length of life. Due to the positive development of mortality it is obvious that for an analysis of mortality is better to use more than one indicator. The aim of this article will be to analyze the development of the modal age at death and compare the results with values of life expectancy for selected European countries. The next part will analyze the differences between these indicators. At the end will be mentioned probable causes of this development.

**Key words:** mortality, life expectancy, modal age at death, Gompertz–Makeham model

**JEL Code**: C02, J11

## Introduction

Current researches show that in most of European populations occurs and there will occur to positive evolution of mortality. It will lead to prolonging of human life. Given this development is good to use (for the analysis of mortality) more of available indicators. In this article we will present two of them: life expectancy at birth and modal age at death (normal length of life).

# 1 Methodology

For the analysis of mortality in the selected European countries will be used life expectancy at birth and modal age at death.

The value of life expectancy is obtained from mortality tables (Cipra, [4]). During the calculation is important to know that data about mortality of the oldest persons may be loaded with systematic and random errors. That's why it is good to smooth and extrapolate calculations for the oldest age groups by some of the available models, which are used for smoothing and extrapolating of mortality curves. In the past, was the most used the Gompertz–Makeham function. Due to this positive evolution, which is clear in some populations there is a change in the evolution of mortality at the highest ages (but for some populations it is true that the Gompertz–Makeham function is still the best).
Gompertz–Makeham function is possible to describe by formula (Boleslawski, Tabeau. [1], Burcin at al. [2], Gompertz, [7]):

$$\mu_x = a + bc^x , \qquad (1)$$

where $\mu_x$ is the intensity of mortality at age $x$, $a$, $b$ and $c$ are parameters from function.

For the intensity of mortality at younger age groups is true the relationship (Fiala, [6]):

$$\mu(x + \frac{1}{2}) = m_x , x=1,2,\ldots,59. \qquad (2)$$

For higher ages is used the Gompertz–Makeham function.

In this article will be used for smoothing and for extrapolating of mortality curves the Gompertz–Makeham function. The calculation of mortality tables will be realized by commonly used algorithm for the calculation of complete mortality tables.

The second very frequently used indicator for description of the evolution of mortality, is the modal age at death (Cipra, [4]). It can be easily interpreted as the age at which the majority of people die. An easier way to estimate it consists in finding the age at which the number of deaths (from table population) is maximal (or the age at which the density of deaths is maximal). In this article will be the normal length of life estimated by using the formula that was derived from the Gompertz–Makeham function based on the formula (Koschin, [9]):

$$\delta(x) = l(x).\mu(x)\,,\ x \in <0;\omega)\,, \tag{3}$$

where δ(x) is the density of deaths, *l(x)* is the number of survivors to age *x*, *μ(x)* is the intensity of mortality and *ω* is the age, at which no one is alive.

For finding the maximum of density of deaths is necessary to solve the equation:

$$\delta'(\hat{x}) = 0\,, \tag{4}$$

where $\hat{x}$ denotes the modal age at death.

After solving the previous equation we obtain the final formula (Koschin, [9]):

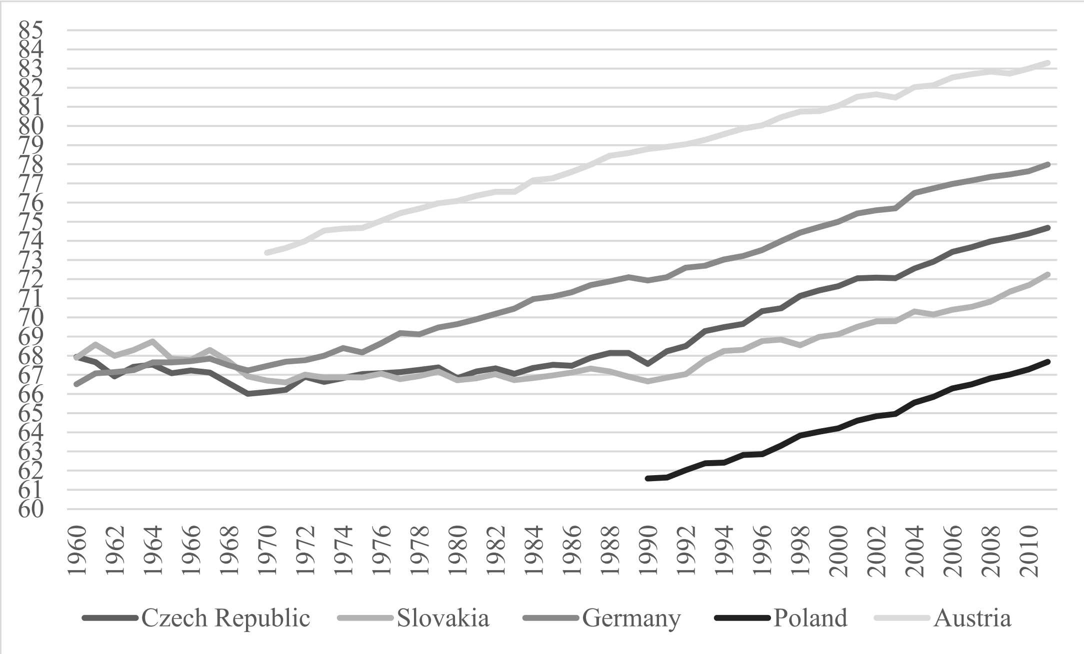
$$\hat{x} = \frac{\ln[\frac{1}{2b}.(\ln c - 2a + \sqrt{(\ln c - 4a).\ln c})]}{\ln c}\,, \tag{5}$$

where *a*, *b*, *c* are parameters from the Gompertz–Makeham function.

## 2 Results

In this paper will be analyze mortality in the Czech Republic and neighboring countries from 1960 to 2010. For the analysis will be used life expectancy at birth and modal age at death. The analysis will be performed for each gender separately.

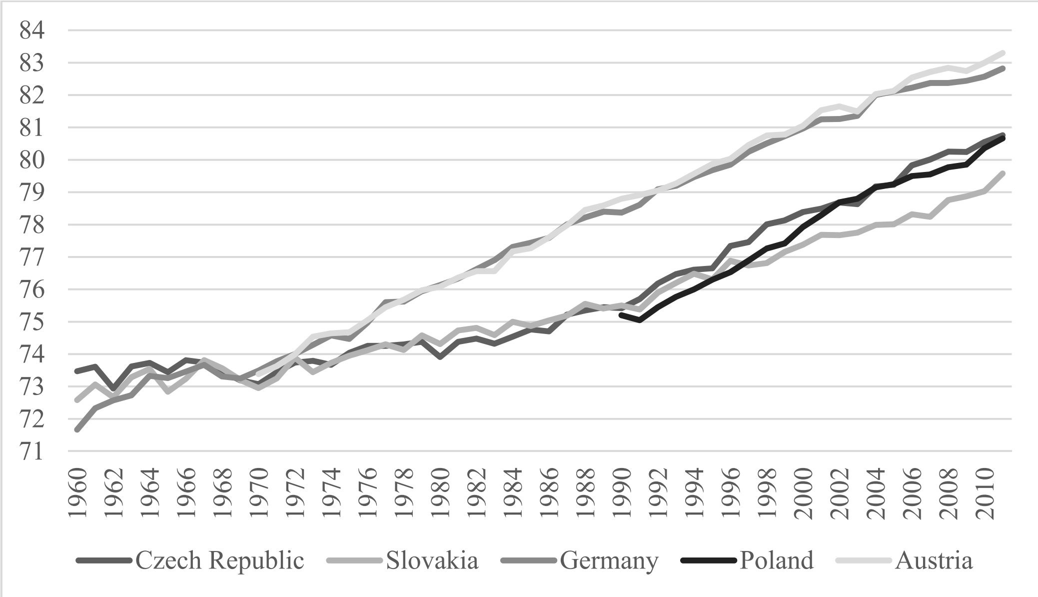
**Fig. 1: Life expectancy in the Czech Republic and neighboring countries - males**




Source: data Eurostat, author's calculation

The first graph shows the evolution of life expectancy at birth in selected European countries. For all populations, it is clear that during the reporting period the value of this indicator has increased. It means positive progress in mortality for each population. In Austria and Germany nearly a sustained increase. Againts in the Czech Republic and in Slovakia is stagnation during the 60th, 70th and 80th years. Sustained growth occurs up to 90 years. If we compare the evolution for individual countries we find that the lowest life expectancy at birth is achieved in Poland and highest one, in Austria. The Czech Republic has the third highest one. For the value of life expectancy at birth in Germany in 2005 was used only estimation of this indicator (the reason is worse quality of data for the year 2005).

**Fig. 2: Life expectancy in the Czech Republic and neighboring countries - females**

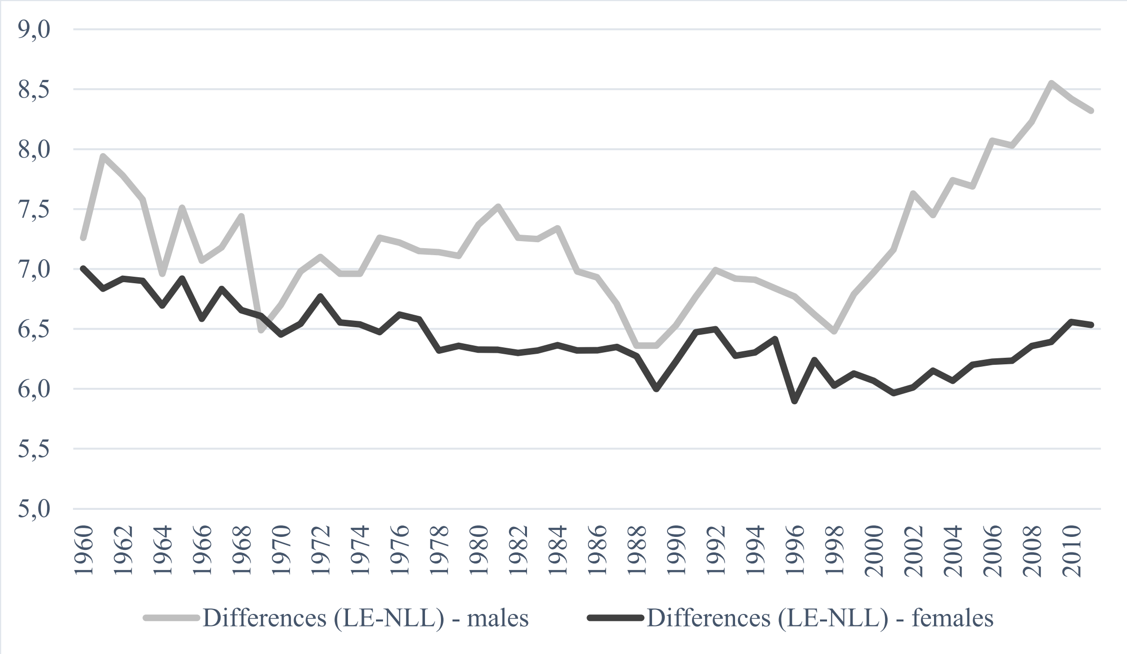



Source: data Eurostat, author's calculation

The second graph shows the evolution of life expectancy at birth for females in selected European countries. As for males, there is a gradual increase in this indicator. The highest value is achieved in Austria and the second highest one is achieved in Germany. As in case of German males, for the value of life expectancy at birth in 2005 was used only estimation of this indicator. Comparing the values obtained for each country, we find that at the beginning of the period were very similar values for each country. Around 1972, the life expectancy has started to increase in Germany and in Austria. At the end of the reporting period, the highest life expectancy at birth is reached in Austria and in Germany. They are followed by the Czech Republic, Poland and Slovakia.

In the following graphs will be compared the differences between the modal age at death and life expectancy at birth. The results will be published separately by gender and they will be compared separately for each country.

**Fig. 3: The differences between modal age at death and life expectancy in the Czech Republic**

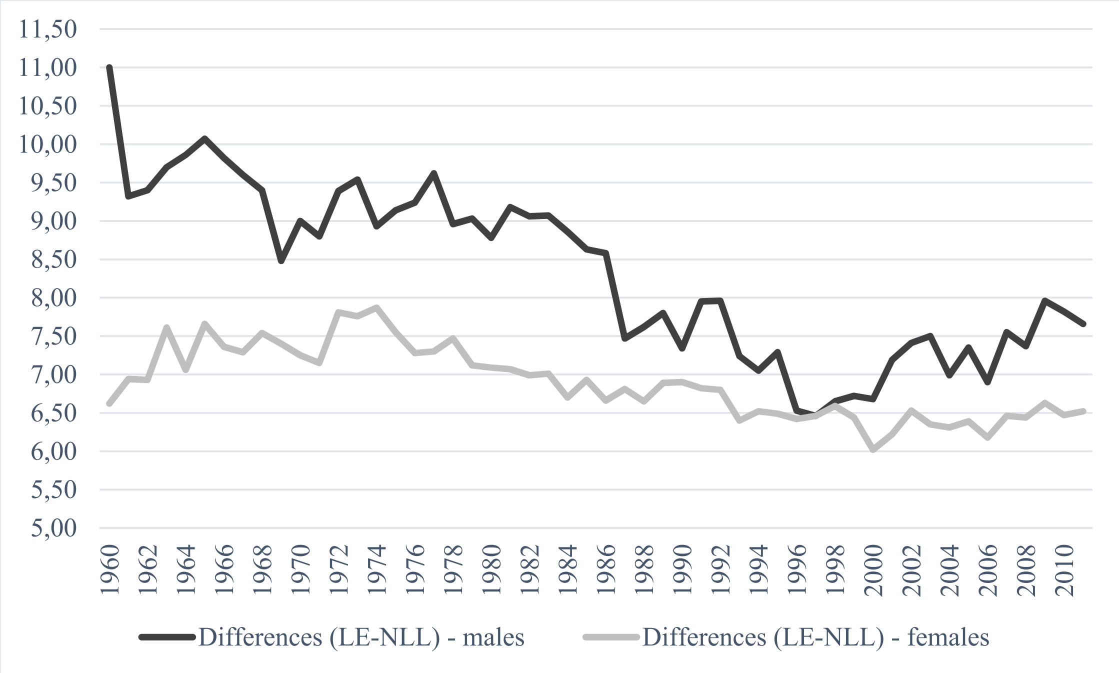



Source: data Eurostat, author's calculation

The third graph shows the differences between the normal length of life and life expectancy at birth in the Czech Republic. If we look at the male's part of the graph, we can see that at the beginning of the period until 1969 basically turns to decrease and increase in the value of the difference. Then the value of the difference grows and begins to decline since 1981. Since 1999, we can see almost continuous growth. The female's part of the graph shows that the value of the differences between these indicators remained almost unchanged. From this we can conclude that modal age at death will show a similar trend as reported life expectancy at birth.

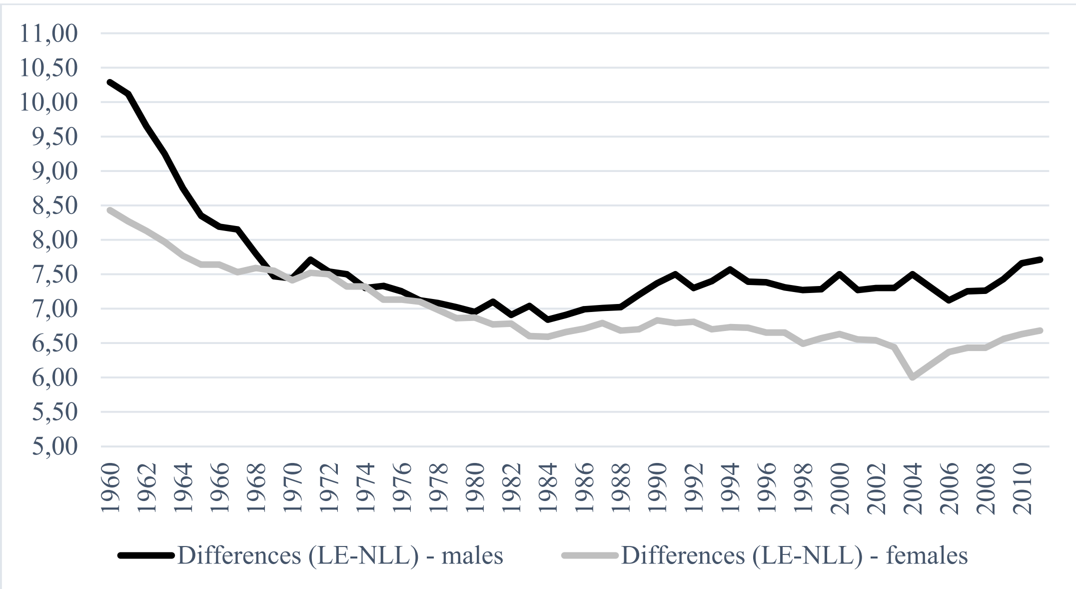
In the fourth graph is shown the evolution of differences between these indicators for males and for females in Slovakia. Comparing the evolution for both sexes, we can find that the evolution for males is much more volatile than for female's population. The highest values of the differences between the modal age at death and life expectancy at birth for Slovak males we could observe at the beginning of the period. It is mainly due to a low value of life expectancy at birth for Slovak boys. Later in the reporting period there is a decline in the value of the difference primarily due to a significant increase in life expectancy at birth, so its value is closer to the value of modal age at death. As was mentioned before for female's population we could observe less volatile evolution. We could even say (when compared the beginning and the end of the period) that it almost did not change the difference between the values of these indicators. The small increase mainly occurred from 1960 to 1974. Then, the difference had decreased and there was no significant change in it later.

**Fig. 4: The differences between modal age at death and life expectancy in Slovakia**




Source: data Eurostat, author's calculations

**Fig. 5: The differences between modal age at death and life expectancy in Germany**




Source: data Eurostat, author's calculation

In the fifth graph are compared the differences between modal age at death and life expectancy at birth in Germany. For males, it is possible to observe significant decline in the value of the difference especially at the beginning of the period (which lasted until 1971) -

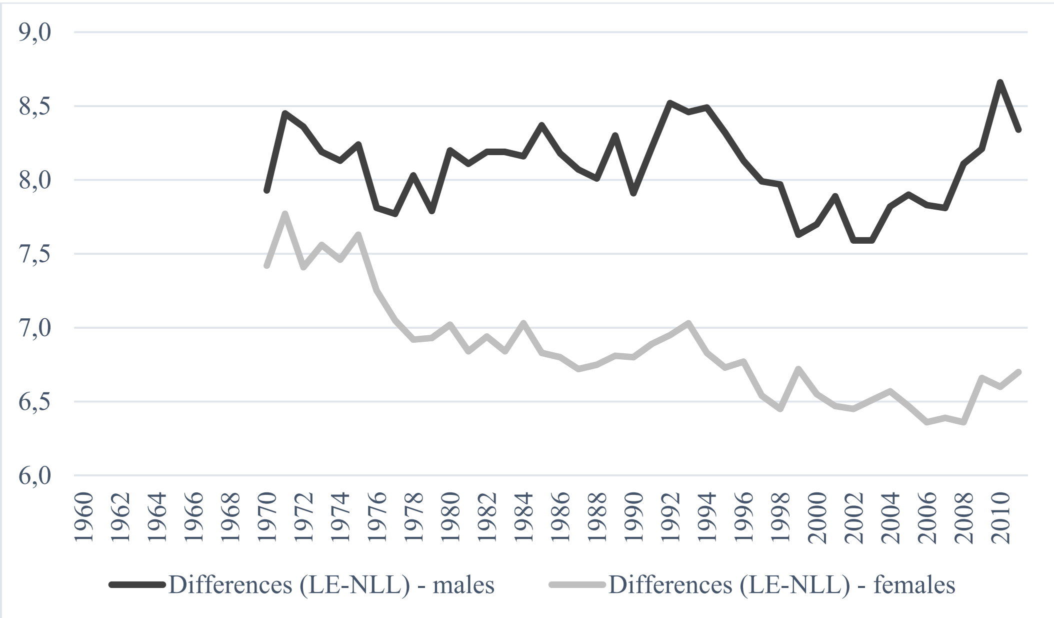
this is mainly due to a decline in the standard life expectancy at the beginning of the period. After that it is a relatively stable evolution. If we focus on females in Germany, we could see that there is only a slight decline. Over the years, there is a slight decline in the value of the difference between the modal age at death and life expectancy at birth.

Graph for polish population will not be published. The reason is pure availability of data.

The sixth graph shows the evolution of the differences between modal age at death and life expectancy at birth in Austria. Even in this population time series are shorter (due to data availability). The development is studied from 1970.

Comparing the evolution obtained for the Austrian males with other male's populations, we can say that there the evolution is least volatile. We can't also say that there has been a decline in the value of the difference or not a significant increase. This will probably caused by a similar evolution of the modal age at death and life expectancy at birth. If we focus on the evolution of the differences between these indicators for Austrian females, we find that during the period there was a decline in the value of the difference.

**Fig. 6: The differences between modal age at death and life expectancy in Austria**




Source: data Eurostat, author's calculations

## Conclusion

From the obtained results it is obvious that in all examined populations, there was a prolongation of human life. It also occurred to extend in the modal age at death (although the increase was so significant). If we take a closer focus on the evolution in individual countries, we find out that the longest living males and females come from Austria. On the contrary, the lowest life expectancy is in Poland (males) and in Slovakia (females).

The investigation of the differences between the modal age at death and life expectancy at birth, we could conclude that the differences for females are less volatile. Conversely males turn there is an increase and then a decline. From the obtained results it is also clear that females essentially no change in the difference between the modal age at death and life expectancy. In contrast, the male's population will its decline (with the exception of the Czech Republic). This evolution of the differences in the population of Czech males is especially caused by rapid growth in modal age at death at the end of the period.

## Acknowledgment

This paper was supported by project VŠE IGA 24/2013 "The mortality and aging population of the Czech Republic".

## References

[1] BOLESLAWSKI, L. – TABEAU, E. Comparing Theoretical Age Patterns of Mortality Beyond the Age of 80. *In: TABEAU, E. – VAN DEN BERG JETHS, A. and HEATHCOTE, CH. (eds.)* 2001. *Forecasting Mortality in Developed Countries: Insights from a Statistical, Demographic and Epidemiological Perspective*, 2001, p. 127 – 155.

[2] BURCIN, B. – TESÁRKOVÁ, K. – ŠÍDLO, L. Nejpoužívanější metody vyrovnávání a extrapolace křivky úmrtnosti a jejich aplikace na českou populaci. Demografie **52**, 2010: 77 – 89.

[3] BURCIN, B. – HULÍKOVÁ-TESÁRKOVÁ, K. – KOMÁNEK, D. *DeRaS: software tool for modelling mortality intensities and life table construction*. Charles University in Prague, 2012, Prague. http://deras.natur.cuni.cz.

[4] CIPRA, T. Matematické metody demografie a pojištění. 1990. Praha, STNL

[5] EUROSTAT. Dostupný z WWW: http://epp.eurostat.ec.europa.eu/.

[6] FIALA, T. *Výpočty aktuárské demografie v tabulkovém procesoru*. Praha: Vysoká škola ekonomická v Praze, 2005.

[7] GOMPERTZ, B.: On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. *Philosophical Transactions of the Royal Society of London* **115** (1825): 513–585.

[8] GAVRILOV, L.A., GAVRILOVA, N.S.: Mortality measurement at advanced ages: a study of social security administration death master file. *North American actuarial journal* **15** (3) (2011): 432 – 447.

[9] KOSCHIN, F. *Aktuárská demografie (úmrtnost a životní pojištění)*. Praha: Vysoká škola ekonomická v Praze, 2000.

[10] MAKEHAM, W.M. On the Law of Mortality and the Construction of Annuity Tables. The Assurance Magazine, and Journal of the Institute of Actuaries 8 (1860): 301–310.

**Contact**

Petra Dotlačilová

University of Economics in Prague

Winston Churchill Sq. 4

130 67, Prague 3

petra.dotlacilova@vse.cz

Jitka Langhamrová

University of Economics in Prague

Winston Churchill Sq. 4

130 67, Prague 3

langhamj@vse.cz