APPLYING MATHEMATICAL MODELS FOR THE EMPLOYEES EVALUATION

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Abstract

Mathematical models are used for staff evaluation. Evaluation of employees is taking an important part of business processes. It is needed that this assessment was carried out exactly also in cases when the subjective evaluation is used.

Key words: Employee evaluation, mathematical modelling, employee satisfaction

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Introduction

Evaluation of employees is an important aspect of controlling the work. The performance of many workers can only be examined in comparison with the satisfaction of their work by colleagues, or subordinates or assigned employees. Implementation of such survey of satisfaction with the performance must be analyzed to determine the level of the evaluators and the importance of their opinion.

1 Evaluation of staff performance

Evaluation of staff is a comprehensive assessment of their working capacity for work use and for their personal and job fulfillment. Systematic evaluation of workers and job outcomes is an important prerequisite for successful work, education tool for strengthening accountability and social relations in the workplace. It would be a tool against inertia, stagnation and mediocrity. (Donnelly et al., 1997)

According to Majtán (2007) staff evaluation deals with:

1. finding attitudes, characteristics, behavior and actions of worker with respect to a particular situation and performance,
2. communication on the results achieved between the evaluated and evaluator,
3. finding how worker performs his work assignments and requirements of superiors,
4. search for ways to improve performance, conduct and compliance measures taken.

The most important functions of staff evaluation:

- ***cognitive*** **-** focuses on exploring the relationship of employee to work, identifying his work attitudes and qualities, work behavior and skills, his personal assumptions and properties.
- ***motivational*** **-** when the evaluation has impact on tangible and intangible valuation of the worker, becomes a source of motivation, personal development and raises the need to increase power output.

There are psychodiagnostic methods that are used to verify the assumptions of individuals to perform a particular function, the finding of intellect, emotion, motivation, characteristics, which may tell about labor discipline, relation to themselves and their surroundings. They are used for recruitment and evaluation of prospective employees of the company. (Drucker, 1992)

The basis of evaluation are also exploratory methods. Their goal is self-assessment of workers, appreciation of their own work, previous life and career. When the worker evaluates himself, there is lower probability of a defensive reaction. On the other hand, self-improvement is more likely. The exploratory methods include curriculum vitae, a personal questionnaire, tests, questionnaires and self-diagnosis.

2 Definition of the model

2.1 Basic elements of the model

In this model, consider a graph G, which consists of a set of edges and a set of vertices, written as follows:

$$G = (V; E) \tag{1}$$

where V denotes the set of vertices and E denotes the set of edges.

Vertices V of a graph G represents evaluators and those that are assessed. These two groups can be distinguished as follows. We distinguish the set of all vertices to two subsets, a subset of A, which represents the evaluators (staff) and a subset of B, which represents those that are assessed (superiors).

Subset of evaluators consists of the following elements $A = \{a_1, a_2, \ldots, a_m\}$. Number of evaluators is therefore m. The second subset of those that are assessed consists of the following elements and $B = \{b_1, b_2, \ldots, b_n\}$. Together we write as

$$V = A \cup B \tag{2}$$

The set of edges E consists of the following parts $E = \{e_1, e_2, \ldots, e_{m \cdot n}\}$. Edges are leading from each vertex of the subset A to each vertex belonging to a subset of B.

At the beginning K points is assigned to the evaluators. These are evenly distributed among m evaluators. Thus, each evaluator will receive the same number of points, namely $\frac{K}{m}$.

On the basis of relations (11) and (13) we define the entire algorithm of one iteration that determine the number of points for the evaluator and evaluated at the end of the p-th iteration.

$$a_{ip} = \sum_{t=1}^{n} \left(wb_{ti} \cdot \sum_{s=1}^{m} \left(a_{i\,p-1} \cdot wa_{is} \right) \right), \text{ for } i = 1, \ldots, m \tag{3}$$

$$b_{ip} = \sum_{s=1}^{m} \left(wa_{si} \cdot \sum_{t=1}^{n} \left(b_{tp-1} \cdot wb_{ti} \right) \right), \text{ for } i = 1, \ldots, n \tag{4}$$

2.2 Defining the fundamental weights

Let us mark the set of fundamental weights Va, Vb respectively. These sets contains $Va = \{va_1, va_2, \ldots, va_n\}$ and $Vb = \{vb_1, vb_2, \ldots, vb_m\}$, where m is the number of elements of a subset of A and therefore the number of evaluators and n is the number of elements of a subset of B and therefore the number of evaluated. Defined basic weight yet must meet the following conditions:

1. The sum of the weights must be equal to one, thus:

$$\sum_{i=1}^{n} va_i = 1 \quad \wedge \quad \sum_{i=1}^{m} vb_i = 1 \tag{5}$$

2. Elements of weight sets Va and Vb must create a non-increasing sequence and therefore must be true:

$$va_i - va_{i+1} \geq 0 \quad pre\, i = \{1, \ldots, n-1\} \tag{6}$$

$$vb_i - vb_{i+1} \geq 0 \quad pre\, i = \{1, \ldots, m-1\} \tag{7}$$

3. For all the basic weights from the sets Va and Vb is valid:

$$va_i \geq 0 \quad pre\,\forall i; i = \{1, \ldots, n\} \tag{8}$$

$$vb_i \geq 0 \quad pre\,\forall i; i = \{1, \ldots, m\} \tag{9}$$

4. If the condition 1 and 3 is met, then necessarily must be true that any element of the set Va or Vb has numeric value less than or equal to 1

$$va_i \leq 1 \quad pre\,\forall i; i = \{1, \ldots, n\} \tag{10}$$

$$vb_i \leq 1 \quad pre\,\forall i; i = \{1, \ldots, m\} \tag{11}$$

2.3 Defining the specific weights of edges

To determine specific weights to be used on a particular edge between the evaluator and the evaluated is dependent on the preferences defined by the assessor to the assessed as well as the use of basic set of weights. Let us denote the system preferences as P. Each evaluator has its own vector of preferences to other evaluated identified as $Pa_i = \{pa_{i1}, pa_{i2}, \ldots, pa_{in}\}$. This vector must yet meet the following conditions:

1. if $s \neq t \Rightarrow pa_{it} \neq pa_{is}$ for $i \in \{1, 2, \ldots, m\}$
2. $pa_{ij} \in \{1, 2, \ldots, n\}, \quad for\ \forall i, i \in \{1, 2, \ldots, m\} \wedge \forall j, j \in \{1, 2, \ldots, n\}$

These preferences are determined based on the subjective attitude of the evaluator to the group of evaluated.

In previous subsections, we defined a set of of basic weights. From this set there is a projection of elements into a set of specific weights of the evaluator, or evaluated. Thus

$$Va \xrightarrow{Pa_i} Wa_i \text{ for } i \in \{1, 2, \ldots, m\} \tag{12}$$

In doing so, the projection is assigned according to preferences. According to the preferences (of order), for a particular combination of assessor and the assessed value of a particular weight is selected.

3 Example of use of the model

In company dealing with heat production in eastern Slovakia, the employee satisfaction survey was conducted with employees of the coordination center. Coordination center staff consists of 5. One employee of coordination center always manages the activities of other staff working in the field of production. Selected section of production consists of 10 employees. These employees work as a service to other departments, and are assigned on the basis of orders from various departments of the company.

Within the evaluation of employees, the company decided to determine qualitative ranking of coordination center staff, thus how other working employees are satisfied with their work. Questionnaires revealed preferences of staff to the coordinator.

We define the set of evaluated staff from the coordination center are the elements of the set B. Set A represents the staff in the field of production. Thus, these sets can be written as follows:

$$\begin{aligned} A &= \{a_1, a_2, \ldots, a_9, a_{10}\} \\ B &= \{b_1, b_2, b_3, b_4, b_5\} \end{aligned} \tag{13}$$

Since on cardinality of individual sets we define the cardinality and structure of basic sets of weights, namely:

$$\begin{aligned} Va &= \{va_1, va_2, va_3, va_4, va_5\} \\ Vb &= \{vb_1, vb_2, \ldots, vb_9, vb_{10}\} \end{aligned} \tag{14}$$

Listed sets of fundamental weights are defined as we stated in section 4.2. We begin by defining a set of values Vas and Vbs. These values are created so that referred sequences are declining. And so

$$Vas = \{50{,}25{,}15{,}6{,}4\} \tag{15}$$

Then we calculate the specific sets of fundamental weights. First, it is necessary to determine the sum of the values. The sum of set Vas is 100. Therefore, each of these values needs to be divided by the sum of the value and thus we obtain the following set of fundamental weights.

$$Va = \{0{,}5;0{,}25;0{,}15;0{,}06;0{,}04\} \tag{16}$$

Due to the large number of elements we do not display here directly the listing of all values of weights for a set of Vbs, or Vb.

To determine specific weights that are used in the model, we need still to determine the system of preferences. Preferences of elements of set A to a set B, thus evaluation of coordinators by individual employees of production unit are determined on the basis of the findings using a questionnaire, which was attended by all employees and therein they had to mark preference directly. So we set the following system of preferences:

$$\begin{aligned}
Pa_1 &= \{1,2,3,4,5\} \\
Pa_2 &= \{1,3,2,5,4\} \\
Pa_3 &= \{2,3,1,4,5\} \\
Pa_4 &= \{3,2,1,4,5\} \\
Pa_5 &= \{4,5,3,1,2\} \\
Pa_6 &= \{5,4,2,3,1\} \\
Pa_7 &= \{4,5,1,3,2\} \\
Pa_8 &= \{1,4,2,3,5\} \\
Pa_9 &= \{3,4,1,2,5\} \\
Pa_{10} &= \{2,4,1,3,5\}
\end{aligned} \tag{17}$$

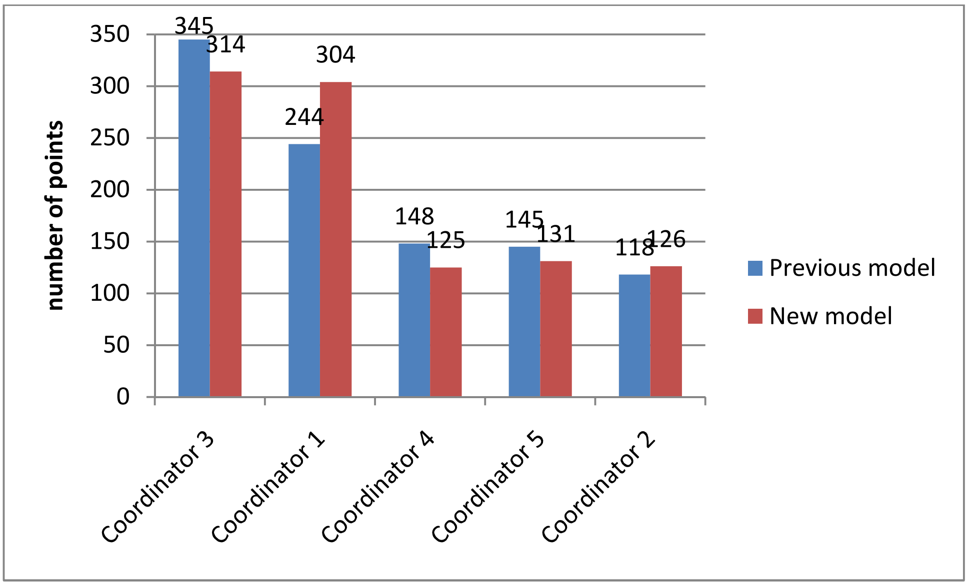
Qualitative evaluation of evaluators themselves can be made on the basis of performance criteria, in which we prefer individual evaluators on the basis of the number of activities. Let us define the following preference, which is uniform for of all evaluated coordinators as a single criterion.

Therefore let us have system of preferences Pb, which is the same for all values. Therefore, we define it as follows

$$\begin{aligned}
Pb_1 &= \{1,2,3,4,5,6,7,8,9,10\} \\
Pb_2 &= \{5,1,4,6,9,2,7,8,3,10\} \\
Pb_3 &= \{3,2,1,7,5,6,4,9,10,8\} \\
Pb_4 &= \{4,3,2,9,5,6,7,1,8,10\} \\
Pb_5 &= \{4,3,2,9,5,6,7,1,8,10\}
\end{aligned} \tag{18}$$

Subsequently we run chosen model. The following chart shows the results of the values in the last iteration. In this illustrative example we let the model iterate for specified number of iterations equal to 20. Iteration does not stop when we reach the value defined in the individual criteria. We define that at the beginning the volume of points was K = 1000.

Fig. 1: Output scores for individual coordinators



Source: own processing

After calculating the value of HHI outcome of occurs. We can see that the order of staff evaluated (coordinators) changed when an employee who was in the original model on third place, found himself in fifth place and the employee originally in the fourth place found himself in the third. At the same time there was also shift of coordinator 2 from the last place to the penultimate. We can see that in the original model between employees 1 and 3, there was a significant difference in the number of points gained, but after using the proposed model, their order was although preserved, but the difference has diminished significantly.

Conclusion

In this paper we dealt with problems of evaluation of employees carried out through peer review of other subordinate staff. We have defined the model for the assessment of employees, which we used in a particular business. We are also comparing this model with the classical method of assessment of such procedures, which are used in enterprises today due to their low economic difficulty.

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